SEMIFREE INVOLUTIONS ON SPHERE KNOTS

Chao-Chu Liang

If m is a positive integer, an m-*knot* (S^{m+2}, Σ^m) consists of an $(m+2)$-homotopy sphere S^{m+2} and an m-homotopy sphere Σ^m differentiably embedded in it. A $(2n-1)$-knot is *simple* if the homotopy groups of its complement $S^{2n+1} - \Sigma^{2n-1}$ coincide with those of the circle in dimension less than n. From now on, we assume $n \geq 2$.

To each simple knot $(S^{2n+1}, \Sigma^{2n-1})$ there corresponds an associated Seifert matrix A (see [1], [3]) such that $A + \varepsilon A^T$ is unimodular, where $\varepsilon = (-1)^n$ and A^T is the transpose of A. The matrix A is determined by a Seifert submanifold V, a $2n$-submanifold of S^{2n+1} that bounds M^{2n-1}; and V can be chosen to be $(n-1)$-connected (see [2]). The normal bundle of Σ^{2n-1} in S^{2n+1} is trivial. Let $Y = \text{closure}(S^{2n+1} - \Sigma^{2n-1} \times D^2)$, and let Y_V be the $(n-1)$-connected manifold obtained by cutting S^{2n+1} along V with $\partial Y_V = V_+ \cup V_-$ (two copies of V). The matrix A is the matrix for the mapping $j_+\colon H_n(V_+) \to H_n(Y_V)$, and the matrix $(-1)^{n+1} A^T$ is the matrix for the mapping $j_-\colon H_n(V_-) \to H_n(Y_V)$, with respect to the bases given by the Alexander duality (see [1], [2]).

Let T be an involution (T^2 = identity) acting differentiably on S^{2n+1} with Σ^{2n-1} as its fixed points, denoted by $(T; S^{2n+1}, \Sigma^{2n-1})$. If $(S^{2n+1}, \Sigma^{2n-1})$ is simple, then the orbit space S^{2n+1}/T is easily seen to be a homotopy sphere, and $(S^{2n+1}/T, \Sigma^{2n-1})$ is again a simple knot. The purpose of this note is to determine which simple knot can be realized as the orbit space of such an involution.

PROPOSITION. *A simple knot* $(S^{2n+1}, \Sigma^{2n-1})$ *is the orbit space of a* $(T; S_1^{2n+1}, \Sigma^{2n-1})$ *if and only if both* $A + A^T$ *and* $A - A^T$ *are unimodular, where* A *is its Seifert matrix determined by some* $(n-1)$*-connected submanifold* V^{2n} *in* S^{2n+1}.

Proof. Let $\{W_i, V_{+i}, V_{-i}\}$ $(i = 1, 2)$ be two copies of $\{Y_V, V_+, V_-\}$. Construct a manifold X by joining W_1 and W_2 by gluing V_{+1} to V_{-1} and V_{+2} to V_{-1}. From the Mayer-Vietoris sequence

$$\to H_{q+1}(X) \to H_q(V_{+1}) \oplus H_q(V_{-1}) \xrightarrow{\lambda_q} H_q(W_1) \oplus H_q(W_2) \to H_q(X) \to$$

we see that

$$H_q(X) = \begin{cases} Z & \text{if } q = 0 \text{ or } q = 1, \\ 0 & \text{if } q \neq n \text{ and } q \neq n+1, \end{cases}$$

and $H_n(X) = H_{n+1}(X) = 0$ if and only if λ_n is unimodular. But

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