BOUNDING A FREE ACTION OF A DIHEDRAL GROUP

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1. RESULTS

Let C be a cyclic group of order 2^{n+1} ($n \geq 1$), and let G be one of the non-abelian split extensions of C by Z_2. The dihedral group of order 2^{n+2} is one example; there are two others [5, p. 187].

This paper considers smooth actions of G preserving a unitary (that is, weakly complex) structure on a smooth manifold. Let $U_*(G)$ be the bordism of all such actions, and let $\hat{U}_*(G)$ be the corresponding bordism of free unitary G-actions. Full definitions can be found in [10]. By [10, Proposition 2.3], we know that $\hat{U}_*(G) \cong U_*(BG)$.

THEOREM. *The kernel of the forgetful homomorphism* $s\colon \hat{U}_*(G) \to U_*(G)$ *is precisely* $\tilde{U}_*(BG)$.

COROLLARY. *Let* $\phi : G \times M \to M$ *be a free unitary* G*-action on a closed manifold. Then* $[M, \phi] = 0$ *in* $U_*(G)$ *if and only if* $[M] = 0$ *in* U_*.

To derive the corollary, one uses the analogue of [4, (19.4)] for unitary actions; this shows that $[M] = 0$ if and only if $[M, \phi] \in \operatorname{im} \tilde{U}_*(BG)$.

It is worth noticing that to prove the theorem for any group, it suffices to establish it for the Sylow subgroups (see [7, Proposition 6]). In particular, our results imply the theorem and corollary for a dihedral group of any order.

2. A TRANSVERSALITY LEMMA

Suppose H is a finite group. Let M and N be smooth H-manifolds, and let $P \subseteq N$ be an invariant submanifold. One says that *transversality* holds for (M, N, P) if, given an equivariant $f\colon M \to N$ and a closed invariant $A \subseteq M$ such that f is transverse to P on A, one may deform f by an H-homotopy making it transverse to P on all of M and leaving f fixed in a neighborhood of A.

LEMMA 1. *Transversality holds for* (M, N, P) *if either*

(a) H *acts freely on* M *or*

(b) H *is nilpotent and the normal bundle* $\nu \to P$ *has the property that, if* $hp = p$ *for some* $h \in H$ *and* $p \in P$, *then* $hx = x$ *for all* $x \in \nu_p$.

Proof. The sufficiency of (a) is fairly well known; a proof is to appear in [8, Proposition 2.2]. The sufficiency of (b) is a generalization of [10, Lemma 4.2]. Since H is nilpotent, it contains a central cyclic subgroup T of prime order. By the argument of [10, Lemma 4.2], we may assume that the fixed set M^T of T is empty.

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