BOUNDS FOR THE COEFFICIENTS OF CYCLOTOMIC POLYNOMIALS

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1. INTRODUCTION

We define the nth cyclotomic polynomial $\Phi_n(z)$ by the equation

$$\Phi_n(z) = \prod_{\substack{r=1 \\ (r,n)=1}}^{n} (z - e(r/n)) \qquad (e(\alpha) = e^{2\pi i\alpha}), \tag{1}$$

and we write

$$\Phi_n(z) = \sum_{m=0}^{\phi(n)} a(m, n)\, z^m, \tag{2}$$

where ϕ is Euler's function. P. T. Bateman [1] has shown that

$$|a(m, n)| < \exp\left(\frac{1}{2}\, d(n) \log n\right), \tag{3}$$

where d is the divisor function. P. Erdös has given two proofs [2], [3] of the existence of a positive number c such that for infinitely many natural numbers n,

$$\log \max_m a(m, n) > \exp\left(\frac{c \log n}{\log \log n}\right). \tag{4}$$

Erdös has asked whether it is possible to take c arbitrarily close to $\log 2$, which would imply that Bateman's result is best possible. In Theorem 1 we give an affirmative answer to this question. In fact, we even show that the choice $c = \log 2$ is permissible.

THEOREM 1. *There are infinitely many natural numbers* n *such that*

$$\log \max_m a(m, n) > \exp\left(\frac{(\log 2)(\log n)}{\log \log n}\right). \tag{5}$$

Erdös and R. C. Vaughan [4] have shown that

$$|a(m, n)| < \exp((\tau^{1/2} + o(1))\, m^{1/2}) \tag{6}$$

uniformly in n as $m \to \infty$, where

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