THE REGULAR RING AND THE MAXIMAL RING OF QUOTIENTS OF A FINITE BAER *-RING

Izidor Hafner

In the first section of this paper we extend the construction of the regular ring **C**, which was defined in the work of S. K. Berberian [1], [2, Chapter 8]. The following theorem, the central result of this article, can be found in the second section. *If* A *is a finite Baer *-ring satisfying the condition* LP ~ RP *and containing sufficiently many projections, then the involution of* A *is extendible to the maximal ring of right quotients* Q *of* A. Next we show that the matrix ring $\mathbf{C}_n$ is also a Baer *-ring. In the last section we discuss the connection with Berberian's construction.

Because of the considerable overlap with the work of E. S. Pyle, Jr. (of which the author was informed after submission of the paper), some of the proofs that can be found in [6] are omitted.

1. CONSTRUCTION OF THE RING C

In this section we follow [2, Chapter 8], where the reader will find the missing definitions and proofs. We assume that A is a finite Baer *-ring satisfying the condition LP ~ RP; that is, the statement LP(x) ~ RP(x) is valid for all $x \in A$. Then A satisfies the parallelogram law (P) and generalized comparability (GC). The lattice of projections in A is a continuous geometry: if D is a directed index set and $\gamma < \delta$ ($\gamma, \delta \in D$) implies $e_\gamma \leq e_\delta$ (e_γ, e_δ are projections in A), and f is a projection in A, then

$$f \cap \sup_{\delta \in D} \{e_\delta\} = \sup_{\delta \in D} \{f \cap e_\delta\}.$$

LEMMA 1. *Let* $\{e_\delta\}$ *be a set of projections in* A, *let* D *be a directed index set, and let* $\gamma < \delta$ *imply* $e_\gamma \leq e_\delta$. *If* $e_\delta \lesssim f$ *for all* $\delta \in D$, *then* $\sup\{e_\delta\} \lesssim f$.

Proof. See [5, p. 115, Hilfssatz 1.5], [2, Section 33, Exercises 1 and 4; Section 34, Exercise 3].

In this section, D denotes a fixed directed index set.

A *strongly dense domain* (SDD) in A is a family of projections $\{e_\delta\}$ such that $\gamma < \delta$ ($\gamma, \delta \in D$) implies $e_\gamma \leq e_\delta$ and $\sup_{\delta \in D} \{e_\delta\} = 1$.

LEMMA 2 [2, p. 213, Lemma 1]. *If* $\{e_\delta\}$ *and* $\{f_\delta\}$ *are* SDD, *then* $\{e_\delta \cap f_\delta\}$ *is an* SDD.

Let $\{e_\delta\}$ be an SDD, and let $x \in A$. Then it can be shown that if $e_\delta x e_\delta = 0$ for all $\delta \in D$, then $x = 0$. Similarly, if $e_\delta x e_\delta$ is self-adjoint for all δ, then x is self-adjoint [2, p. 218, Exercise 2].

An *operator with closure* (OWC) is a pair of sequences (x_δ, e_δ), where $x_\delta \in A$ and $\{e_\delta\}$ is an SDD, such that $\gamma < \delta$ implies $x_\delta e_\gamma = x_\gamma e_\gamma$ and $x_\delta^* e_\gamma = x_\gamma^* e_\gamma$.

Received February 23, 1974.

Michigan Math. J. 21 (1974).