FINITE GROUPS OF R-AUTOMORPHISMS OF R[[X]]

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Let R be an integral domain with identity, let X be an indeterminate over R, let S be the formal power series ring R[[X]], and let G be a finite group of R-automorphisms of S. If $S^G = \{h \in S \mid \phi(h) = h \ \forall \phi \in G\}$, then we call S^G the *ring of invariants* of G. In [10], P. Samuel shows that if R is a local domain (that is, a Noetherian integral domain with unique maximal ideal M) and R is complete in the M-adic topology, then there exists a $f \in S$ such that $S^G = R[[f]]$.

In recent papers, O'Malley [7] and J.-B. Castillon [1], [2] have considered the same problem. O'Malley shows that the same conclusion holds if R is a Noetherian integral domain with identity whose integral closure is a finite R-module. In [1], using simpler techniques than either Samuel or O'Malley, Castillon extends Samuel's result to the case when R is a quasi-local domain that is a complete Hausdorff space in its maximal ideal-adic topology. In [2], using the results of this author [6], [7], and [8], Castillon proves that $S^G = R[[f]]$ if R is a Noetherian integral domain with identity. The specific results of [2] are contained in Theorem 5 and the corollary of this paper.

In this paper, we prove the following more general result.

THEOREM 1. *Let* R *be an integral domain with identity, let* X *be an indeterminate over* R, *let* S *be the formal power series ring* R[[X]], *and let* G *be a finite group of* R-*automorphisms of* S. *If* $f = \prod_{\phi \in G} \phi(x)$ *and* S^G *denotes the ring of invariants of* G, *then* $S^G = R[[f]]$.

We make strong use of Theorem 2.6 of [8] and Corollary 5.8 of [6]. In Section 2, observing an easy extension of a proof given in [1], we derive a result (Theorem 4) of prime importance in our proof of Theorem 1. In Section 3, we prove Theorem 1.

1. NOTATION AND TERMINOLOGY

All rings considered in this paper are assumed to be commutative and to contain an identity element. We use the symbols ω and ω_0 to denote the sets of positive and nonnegative integers, respectively, and the symbols $\subseteq$ and $\subset$ to denote containment and proper containment, respectively. If R is a ring, then J(R) will denote the Jacobson radical of R, and S will denote the formal power series ring R[[X]]. If $g = \sum_{i=0}^{\infty} c_i X^i$ is a nonzero element of S such that the first nonzero coefficient of g is c_k, then we say g *has order* k, and we write $O(g) = k$. If d is an element of R, then (d) will denote the ideal of R generated by d.

If A is an ideal of R, then the collection $\{A^k\}_{k \in \omega}$ of ideals of R induces a topology, called the A-*adic topology*, on R. We write (R, A) to denote the topological ring R under this topology. It is well known that (R, A) is a Hausdorff space if

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Received March 3, 1973.
This research was supported by a University of Houston Faculty Research Grant.

Michigan Math. J. 20 (1973).