# ON HOMOTOPY SEVEN-SPHERES THAT ADMIT DIFFERENTIABLE PSEUDO-FREE CIRCLE ACTIONS

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## 1. INTRODUCTION

This paper treats differentiable pseudo-free circle actions on homotopy 7-spheres. In an earlier paper, we showed how to construct all of these actions, but left open the question which homotopy 7-spheres can occur [3]. This question is now answered by the following theorem.

THEOREM A. *Each of the* 28 *homotopy* 7*-spheres admits a differentiable pseudo-free circle action with exactly one exceptional orbit.*

We note that only 10 of 28 homotopy 7-spheres admit differentiable free circle actions [2].

## 2. CONSTRUCTION OF SOME ASSOCIATED MANIFOLDS

A differentiable action of the circle group $G$ on a homotopy 7-sphere $\Sigma^7$ is said to be *pseudo-free* if it is an effective action for which every isotropy group is finite and the set of exceptional orbits (that is, the set of orbits where the isotropy group is not trivial) is finite but not void. Suppose that such an action is given, and let

$$Gb_1, \cdots, Gb_k$$

be the exceptional orbits in $\Sigma^7$. For $i = 1, \cdots, k$, the isotropy group $G_{b_i}$ at $b_i$ is a finite cyclic group $\mathbb{Z}_{q_i}$ of order $q_i$, where $q_i$ is an integer greater than 1, and since $\Sigma^7$ has the integral homology of a 7-sphere, we see that the integers $q_1, \cdots, q_k$ are mutually relatively prime. We let

$$q = q_1 \cdots q_k.$$

In the following, $\mathbb{C}^n$ denotes the unitary n-space, $D^{2n}$ denotes the closed unit (2n)-disk in $\mathbb{C}^n$, and $S^{2n-1}$ denotes the boundary of $D^{2n}$, that is, the unit (2n - 1)-sphere in $\mathbb{C}^n$. Then

$$G = S^1,$$

and the orthogonal action of $G$ on $S^7$ given by

$$g(z_1, z_2, z_3, z_4) = (g^q z_1, gz_2, gz_3, gz_4)$$

is pseudo-free and has exactly one exceptional orbit $Gb$, where $b = (1, 0, 0, 0)$ and $G_b = \mathbb{Z}_q$.

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