# COMMUTANTS OF SHIFTS ON BANACH SPACES

Richard M. Crownover

Among the operators on an arbitrary complex Banach space, we consider a class of operators that can be represented as certain unilateral shift operators on a Banach space of sequences. We associate an analytic structure with each of these operators in such a way that each element of the Banach space may be expressed as an analytic function on a neighborhood in the spectrum of the operator. This identification enables us to view the commutant of the operator as an algebra of multiplications by bounded analytic functions, thus giving us a commutant theory similar to that for the unilateral shift on the sequence space $\ell^2$ [5, Problem 116].

For the special case in which the shift operator is an isometry, sharper estimates on the size of the convergence set for the commutant are available than for the general case. We show that in this special case, the commutant can be identified with a subalgebra of the space $H^\infty$ on the unit disk.

We apply the theory developed to a commutant problem of A. L. Shields and L. J. Wallen [12], and we make other applications to results on factorization of power series with coefficients in $\ell^p$, the spectrum of a shift, and a question of existence of roots. In the last section, we discuss special cases of an approximation theory for elements of the commutant.

In addition to the work of Shields and Wallen on commutant problems, we mention the work of R. Gellar [2], [3], which has some structural similarity to the present work. Gellar also utilizes power series and considers commutant problems. The main distinction, however, is that Gellar starts with a particular Schauder basis and considers weighted shifts with respect to that basis. These shifts are occasionally included in the collection of shifts considered in our work. Moreover, a large collection of the shifts considered here are not of the type considered by Gellar. A further distinction is that our definition of a shift is basis-free, and indeed we carry out our work independent of the existence of a Schauder basis.

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## 1. PRELIMINARIES

A standard definition is that a unilateral shift on a Hilbert space is an operator U for which there exists an orthonormal basis $\{e_n\}_{n=1}^{\infty}$ such that $Ue_n = e_{n+1}$ for $n = 1, 2, 3, \cdots$. Unilateral shifts on Hilbert space have been extensively studied, and many of their properties are well known ([5, Chapter 14] and [6, Chapter 7]). Much of the utility of such an operator derives from its unitary equivalence to the operator $S_z$ of multiplication by z on the Hilbert space of square-summable power series $\sum_{n=0}^{\infty} a_n z^n$ (the classical Hardy space $H^2$). Thus an inherent analytic structure is associated with the unilateral shift, and this structure facilitates the study of a

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