RINGS OF TYPE II

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1. INTRODUCTION

We define a *ring of type* II to be an integral domain R with the following properties:

(1) R is a complete, Noetherian local ring of Krull dimension one.

(2) Every ideal of R can be generated by two elements.

Of course, a complete discrete valuation ring is a ring of type II. But not all rings of type II are valuation ring (example: the ring of all formal power series in one variable over a field, with the linear term missing). It is the purpose of this paper to characterize rings of type II in terms of a Hausdorff condition and the structure of certain modules.

Definition. An integral domain R is said to have *property* D if every torsion-free R-module of finite rank is a direct sum of R-modules of rank 1.

Definition. The *Krull dimension* of an integral domain is the maximal number of terms in a chain of nonzero prime ideals.

In [4] we proved the following theorem.

THEOREM 1 [4, Theorem 4]. *If* R *is an integral domain, the following statements are equivalent.*

(1) R *is a ring of type* II.

(2) R *is a Noetherian integral domain with property* D.

The aim of this paper is to replace the Noetherian assumption with the weaker Hausdorff assumption that $\bigcap I^n = 0$ for every proper ideal I of R. We shall prove the following theorem (see Section 4):

THEOREM 11. *If* R *is an integral domain, the following statements are equivalent.*

(1) R *is a ring of type* II.

(2) R *has property* D, *and* $\bigcap I^n = 0$ *for every proper ideal* I *of* R.

2. REVIEW

Definition. An integral domain R is said to have a *remote quotient field* Q if there exists an R-module S such that $R \subset S \subsetneq Q$ and $S^{-1} = 0$, where $S^{-1} = \{x \in Q \mid xS \subset R\}$.

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