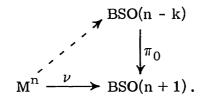
SECTIONING BUNDLES OF HIGH FILTRATION AND IMMERSIONS

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Let M^n be a closed orientable manifold of dimension n, and let ν be its stable normal bundle. M. W. Hirsch [7] has shown that if k < n, then M^n immerses in R^{2n-k} if and only if the geometric dimension of ν is at most n-k. Classifying ν by a map $\nu \colon M^n \to BSO(n+1)$, we find that an immersion is equivalent to a lifting



If k < n/2, then the obstruction theory developed by M. Mahowald [8], [4] and E. Thomas [14], [15] can be applied. This involves trying to compute the obstructions with higher-order cohomology operations, and the method becomes unwieldy for large k, because the construction of operations of order greater than 2 is difficult. In this note we show that if BSO is replaced by its k-connected covering, then the obstructions to lifting any bundle of filtration k+1 can be expressed in terms of higher-order operations that are defined on a generalized cohomology theory $H^*(-; \mathfrak{X})$. The spectrum \mathfrak{X} is simple enough so that these operations can be computed for the normal bundle, and we prove the following result.

THEOREM. Let M^n be a closed orientable manifold of dimension n, and let k be an integer such that 2k < n. If

- (i) M is (k 2)-connected and
- (ii) the normal bundle ν of M is trivial over the k-skeleton, then M^n immerses in R^{2n-k} if and only if $w_{n-k+1}(\nu)=0$.

A. Haefliger and M. W. Hirsch [6] have shown that condition (i) gives an immersion in \mathbb{R}^{2n-k+1} if and only if $w_{n-k+1}(\nu)=0$. J. Becker [2] has proved our theorem with a condition slightly weaker than (ii), namely, that ν is fibre-homotopy trivial over the k-skeleton. These results apply only to immersions however, while the techniques used to prove our theorem apply to the problem of sectioning any bundle.

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