SECTIONING BUNDLES OF HIGH FILTRATION AND IMMERSIONS

John Van Eps

Let M^n be a closed orientable manifold of dimension n, and let ν be its stable normal bundle. M. W. Hirsch [7] has shown that if $k < n$, then M^n immerses in R^{2n-k} if and only if the geometric dimension of ν is at most $n - k$. Classifying ν by a map $\nu: M^n \to BSO(n+1)$, we find that an immersion is equivalent to a lifting

$$\begin{array}{ccc} & & BSO(n-k) \\ & \nearrow & \downarrow \pi_0 \\ M^n & \xrightarrow{\nu} & BSO(n+1). \end{array}$$

If $k < n/2$, then the obstruction theory developed by M. Mahowald [8], [4] and E. Thomas [14], [15] can be applied. This involves trying to compute the obstructions with higher-order cohomology operations, and the method becomes unwieldy for large k, because the construction of operations of order greater than 2 is difficult. In this note we show that if BSO is replaced by its k-connected covering, then the obstructions to lifting any bundle of filtration $k + 1$ can be expressed in terms of higher-order operations that are defined on a generalized cohomology theory $H^*(-; \mathfrak{X})$. The spectrum $\mathfrak{X}$ is simple enough so that these operations can be computed for the normal bundle, and we prove the following result.

THEOREM. *Let* M^n *be a closed orientable manifold of dimension* n, *and let* k *be an integer such that* $2k < n$. *If*

(i) M *is* $(k-2)$*-connected and*

(ii) *the normal bundle* ν *of* M *is trivial over the* k*-skeleton,*

then M^n *immerses in* R^{2n-k} *if and only if* $w_{n-k+1}(\nu) = 0$.

A. Haefliger and M. W. Hirsch [6] have shown that condition (i) gives an immersion in R^{2n-k+1} if and only if $w_{n-k+1}(\nu) = 0$. J. Becker [2] has proved our theorem with a condition slightly weaker than (ii), namely, that ν is fibre-homotopy trivial over the k-skeleton. These results apply only to immersions however, while the techniques used to prove our theorem apply to the problem of sectioning any bundle.

This work is related to my thesis, and I would like to thank my advisor, Professor P. E. Thomas, for his generous help and encouragement. I would also like to thank Professor J.-P. Meyer for reading a draft of this paper and for making several suggestions.

Received January 20, 1971.

This research was partially sponsored by the United States Army under Contract No. DA-31-124-ARO(D)-176.

Michigan Math. J. 19 (1972).