ON SUBNORMAL SUBGROUPS OF FUNDAMENTAL GROUPS OF CERTAIN 3-MANIFOLDS

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Let M be a P^2-irreducible 3-manifold. In [2], it is shown that if F is a 2-sided, closed, incompressible surface in M such that $i_*\pi_1(F)$ is normal in $\pi_1(M)$, then M is either a fibre bundle over S^1 with fibre F, or a line bundle over a closed surface G (and F is then parallel to ∂M), or a union of two such line bundles. In particular, if $\partial M \neq \emptyset$ and $i_*\pi_1(F)$ is normal in $\pi_1(M)$, then $i_*\pi_1(F)$ is of index 2 or 1 in $\pi_1(M)$. In this paper, we show that the same result holds if we replace "normal" by "subnormal." Hence $i_*\pi_1(F)$ is subnormal in $\pi_1(M)$ if and only if it is normal in $\pi_1(M)$. An analogous result holds for noncontractible, simple closed curves in 2-manifolds. By way of an application, we classify the sufficiently large 3-manifolds that have fundamental groups each of whose subgroups is subnormal.

We work throughout in the piecewise linear category. "A surface $F \subset M$" always means a 2-sided embedded surface in M. We say that F is *incompressible* in M if genus$(F) \geq 1$ and $\ker(i_*\pi_1(F) \to \pi_1(M)) = 0$, where i: $F \to M$ denotes inclusion. A 3-manifold is called P^2-*irreducible* if M is irreducible and contains no (2-sided) projective planes.

A subgroup S of a group G is called *subnormal* (in G) if there exists a finite sequence of subgroups $S_1, \cdots, S_n$ of G such that $S \lhd S_1 \lhd \cdots S_n \lhd G$.

I wish to thank John Hempel and John Ledlie for helpful conversations.

1. SUBNORMAL SUBGROUPS OF $\pi_1(M)$

If F is a surface in M, let $i_*: \pi_1(F) \to \pi_1(M)$ denote the homomorphism induced by inclusion.

THEOREM 1. *Let* M *be a compact,* P^2-*irreducible* 3-*manifold, and suppose* F *is a* 2-*sided, closed, incompressible surface in* M *such that* $i_*\pi_1(F)$ *is subnormal in* $\pi_1(M)$. *Then one of the following holds:*

(a) M *is a fibre bundle over* S^1 *with fiber* F.

(b) $M \cong F \times I$.

(c) M *is a twisted line bundle over a closed surface* G, *and* F *is parallel to* ∂M.

(d) F *separates* M *into two twisted line bundles of type* (c).

LEMMA 1. *If* S *is subnormal in* G *and* $U \subseteq G$ *is a subgroup containing* S, *then* S *is subnormal in* U.

Proof. We have subgroups $S_1, \cdots, S_n$ of G such that $S \lhd S_1 \lhd \cdots S_n \lhd G$. Forming intersections with U, we obtain the sequence $S \lhd S_1 \cap U \lhd \cdots \lhd S_n \cap U \lhd U$, and this proves the lemma.

Received April 15, 1970.

Michigan Math. J. 18 (1971).