BEHAVIOR OF NORMAL MEROMORPHIC FUNCTIONS ON THE MAXIMAL IDEAL SPACE OF H^∞

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Many theorems about bounded holomorphic functions hold also in the larger class of normal meromorphic functions. We recall that bounded holomorphic functions can be extended continuously to the maximal ideal space M of H^∞. The main purpose of this paper is to point out that a (meromorphic) function is normal if and only if it admits a continuous extension to the set G of "regular points" of M. In fact, it turns out that if f is meromorphic, then such extensions are actually meromorphic on G. This result is sharp; we present an example of a normal meromorphic function f that cannot be extended continuously to any nonregular point. An examination of this function f yields a new proof that the nonregular points are rare in the sense that they constitute a closed, nowhere dense set [11, p. 102].

K. Stroyan has pointed out to us that the extendibility mentioned above can be established by means of the nonstandard characterization of the Gleason parts of M obtained by M. F. Behrens (unpublished).

1. PRELIMINARIES

We shall consider functions that are defined in the unit disc D with the non-Euclidean hyperbolic metric ρ, and that take their values on the Riemann sphere Ω endowed with the chordal metric χ. The hyperbolic distance $\rho(z, z')$ and the pseudohyperbolic distance $\psi(z, z')$ are defined by

$$\psi(z, z') = \left|\frac{z - z'}{1 - \bar{z}' z}\right| = \tanh[\rho(z, z')].$$

LEMMA 1 (Pick; see [9, p. 239]). *Suppose* f *is holomorphic and bounded by* 1 *in* D. *Then*

$$\rho(f(z), f(z')) \leq \rho(z, z'),$$

for all $z, z' \in D$.

For subsets S and T of D, we define the three pseudometrics

a) $H_\rho(S, T) = \inf\{\varepsilon: S \subset \{z: \rho(z, T) < \varepsilon\},\ T \subset \{z: \rho(z, S) < \varepsilon\}\}$,

b) $\sigma(S, T) = \inf\limits_r H_\rho(S \cap \{|z| > r\},\ T \cap \{|z| > r\})$,

c) $\lambda(S, T) = \inf\limits_r \rho(S \cap \{|z| > r\},\ T \cap \{|z| > r\})$;

the first is called the non-Euclidean Hausdorff pseudometric.

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