COMPACT, CONTRACTIBLE n-MANIFOLDS AND THEIR BOUNDARIES

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The purpose of this paper is to show that for $n \geq 6$ the function, assigning M to ∂M, from the set of compact, contractible n-manifolds to the set of homology $(n-1)$-spheres is a bijection. (A similar result has been announced by M. Kato, see [5].) We also show that for M as above, the group of concordance classes of homeomorphisms of M onto itself is isomorphic to the group of concordance classes of homeomorphisms of ∂M onto itself.

We consider both PL and topological manifolds and maps in this paper. The term manifold allows the possibility that the boundary is not empty. We use ∂M to denote the boundary of a manifold M and int M to denote the interior of M. We use D^n and S^n to denote the standard n-cell and n-sphere. By the term disk we mean a 2-cell. We use $A * B$ to denote the join of spaces A and B. If M is a manifold and P a subpolyhedron of M, then $N(P, M)$ denotes a regular neighborhood of P in M, see J. F. P. Hudson and E. C. Zeeman [4]. Finally, if M and N are manifolds and $h: \partial M \to \partial N$ is a homeomorphism, we denote by $M \cup_h N$ the manifold obtained by identifying $x \in \partial M$ with $h(x) \in \partial N$. We let ρ_M and ρ_N denote the inclusions of M and N, respectively, into $M \cup_h N$. Hence

$$\rho_N^{-1} \circ \rho_M \mid \partial M = h.$$

Furthermore, if $C \subseteq M$ and $C' \subseteq N$, then $C \cup_h C'$ denotes $\rho_M(C) \cup \rho_N(C')$. Although C and C' may also be manifolds with boundary, no confusion should arise between the two uses of the notation $\cup_h$. We shall also not distinguish between $A \subseteq M$ and $\rho_M(A) \subseteq M \cup_h N$ when no confusion can arise.

LEMMA 1. *Let* M *and* N *be contractible* PL *n-manifolds* $(n \geq 5)$. *Let* $h: \partial M \to \partial N$ *be a homeomorphism,* $J \subseteq \partial M$ *a simple closed curve,* $D \subseteq M$ *a disk such that* $D \cap \partial M = J$. *Suppose* T *is a regular neighborhood of* J *in* ∂M, *and let* C *be a regular neighborhood of* $D \cup T$ *in* M, *relative to* $c\ell(\partial M - T)$. *Moreover,* D' *denotes a disk in* N *such that* $D' \cap \partial N = h(J)$, *and* C' *denotes a regular neighborhood of* $D' \cup h(T)$ *in* N, *relative to* $c\ell(\partial N - h(T))$. *Then there exists a* PL *homeomorphism*

$$f: (C \cup_h C', T, D \cup_h D') \to (S^2 \times D^{n-2}, S^1 \times D^{n-2}, S^2),$$

where $C \cup_h C'$ *and* $D \cup_h D'$ *are subsets of* $M \cup_h N$.

Proof. Consider $S = M \cup_h N$. By Van Kampen's theorem, S is simply connected. By the Mayer-Vietoris sequence, S has the same homology groups as ∂M. From the Lefschetz duality theorem [2], we obtain the following diagram:

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