SETS OF UNIQUENESS ON THE PRODUCT OF COMPACT GROUPS

J. E. Coury

Let $X = \prod_{n=1}^{\infty} X_n$ be the product of countably many non-Abelian, compact, topological groups, and let μ denote Haar measure on X. Let f_n be a coordinate function of $V^{(n)}$, where $V^{(n)}$, different from 1, is a continuous, unitary, irreducible representation (CUIR) of X_n. For $\bar{x} = (x_1, x_2, \cdots)$ in X, define $f_n(\bar{x})$ to be $f_n(x_n)$.

A subset C of X is called a *set of uniqueness* with respect to a regular method S of summability (or briefly, a U_S-set) if S-summability to 0 on the complement of C of a series $\sum c_n f_n$ with complex coefficients c_n implies that $c_n = 0$ for each n. Otherwise, C is called a *set of multiplicity* (an M_S-set).

Let d_n be the dimension of the representation space of $V^{(n)}$, and set $M = \sup\{d_n : n \geq 1\}$. We prove that if $M < \infty$ and $\mu(C) < 1/2M$, then C is a U_S-set. If $M = \infty$, then every subset of X of measure 0 is a set of uniqueness. We also demonstrate that if each X_n is connected, then every subset of X of measure less than 1/2 is a U_S-set.

1. PRELIMINARIES

Let μ_n and μ denote normalized Haar measure on X_n and X, respectively, and write the identity element of X as $\bar{e} = (e_1, e_2, \cdots)$, where e_n is the identity in X_n.

For each n, choose α_n in X_n so that there exists a continuous, unitary, irreducible representation $V^{(n)}$ of X_n, on a Hilbert space H_n of dimension $d_n \geq 2$, for which $V^{(n)}_{\alpha_n} \neq I$. (That such a choice is possible in every compact non-Abelian group G can be demonstrated as follows. Let $a, b \in G$ be such that $ab \neq ba$; then $aba^{-1}b^{-1}$ is not the identity in G. By [2, (22.12)], we can find a CUIR V of G such that $V_{aba^{-1}b^{-1}} \neq I$. If V were one-dimensional, we could conclude that $V_{aba^{-1}b^{-1}} = V_a V_b V_a^{-1} V_b^{-1} = I$, contrary to our choice of V.) It follows that $\alpha_n \neq e_n$.

For $V^{(n)}$, let $\{\xi_1^{(n)}, \cdots, \xi_{d_n}^{(n)}\}$ be an orthonormal basis of H_n such that

$$V_{\alpha_n}^{(n)} \xi_k^{(n)} = \lambda_k^{(n)} \xi_k^{(n)} \quad \text{for } k = 1, 2, \cdots, d_n,$$

where $|\lambda_k^{(n)}| = 1$. Since $V^{(n)} \neq I$, there exists an element $q \in \{1, 2, \cdots, d_n\}$ for which $\lambda_q^{(n)} \neq 1$. For such a q and arbitrary $p \neq q$, define the complex-valued

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