ON THE COMPLETENESS OF NULLITY FOLIATIONS

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Several authors have studied the null-space distribution of intrinsic or extrinsic curvature tensors on a Riemannian manifold M (see the references below). That these distributions are integrable and their integral manifolds are totally geodesic is an easy consequence of Bianchi's identity or of the equation of Codazzi and Mainardi, respectively. In several cases, the integral manifolds have been shown to be complete if M is complete (see [1], [5], and [6]). On the other hand, in the study of curvature-like tensor fields or Riemannian double forms, A. Gray ([3] and [4]) assumed the curvature tensors to be recurrent, in order to obtain completeness. It is the aim of the present paper to eliminate this rather strong condition, and at the same time to simplify the previously known proofs of the completeness of the k-nullity and relative-nullity foliations.

The notion of Riemannian double forms with values in a vector bundle seems to be the proper setting for a simultaneous treatment of the extrinsic and the intrinsic case. We refer to [1], [2], and [4] for examples and geometric applications.

Notation and Assumptions. Let $(M, \langle \cdots, \cdots \rangle)$ be a Riemannian manifold with Levi-Civita covariant derivative $\overset{M}{\nabla}$, and let ξ be a vector bundle over M with covariant derivative $\overset{\xi}{\nabla}$. For each integer $p \geq 0$ and each vector bundle η over M, let $\Lambda^p(\eta)$ denote the bundle of alternating p-forms on M with values in η. We are particularly interested in the bundle $\delta_{p,q}(\xi) = \Lambda^p(\Lambda^q(\xi))$, the bundle of *double forms of type* (p, q) *with values in* ξ. Note that in the usual way $\overset{M}{\nabla}$ and $\overset{\xi}{\nabla}$ induce a covariant derivative $\overset{p,q}{\nabla}$ for $\delta_{p,q}(\xi)$. From now on we shall denote all occurring covariant derivatives simply by ∇.

For a double form $A \in \Gamma\delta_{p,q}(\xi)$, define $A^* \in \Gamma\delta_{p+1,q-1}(\xi)$ and $DA \in \Gamma\delta_{p+1,q}(\xi)$ by the equations

$$A^*(X_0, \cdots, X_p)(Y_2, \cdots, Y_q) = \sum_{j=0}^{p} (-1)^j A(X_0, \cdots, \hat{X}_j, \cdots, X_p)(X_j, Y_2, \cdots, Y_p)$$

and

$$(DA)(X_0, \cdots, X_p) = \sum_{j=0}^{p} (-1)^j (\nabla_{X_j} A)(X_0, \cdots, \hat{X}_j, \cdots, X_p),$$

for all vector fields $X_0, \cdots, X_p, Y_2, \cdots, Y_p$ on M.

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