A RELATION BETWEEN POINCARÉ DUALITY AND QUOTIENTS OF COHOMOLOGY MANIFOLDS

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1. INTRODUCTION

A *proper map* is one for which the inverse image of each compact set is compact. Such a map is said to be *acyclic* over a coefficient domain L if each point-inverse is cohomologically trivial over L. Unless we say otherwise, we assume that L is an arbitrary but fixed principal ideal domain. We use the sheaf-theoretic cohomology theory and the homology theory defined by A. Borel and J. C. Moore as explicated in [3]. All supports for these theories are closed unless "c" appears as a subscript or superscript, in which case compact supports are to be taken. We replace "n-dimensional cohomology manifold" by the acronym "n-cm."

K. W. Kwun and F. Raymond [4] have proved the following result. Suppose that X is a compact, connected, orientable n-cm, and that Y is an n-cm. In addition, suppose that

$$f\colon (X, A) \to (Y, B) \qquad (A \neq X)$$

maps X - A onto Y - B and maps the closed set A onto B such that $f \mid X - A$ is acyclic. Then A satisfies a condition resembling Poincaré duality. More precisely, for $p \neq 0$ and $p \neq n$, the homomorphism

$$\phi\colon H_{n-p}(A) \xrightarrow{i_*} H_{n-p}(X) \xrightarrow{\Delta} H^p(X) \xrightarrow{i^*} H^p(A)$$

is an isomorphism, where i_*, i^* are induced by inclusion and Δ is the Poincaré duality isomorphism.

Theorem 1 of this paper provides a converse to the result of Kwun and Raymond in the case where B is a point and A is a continuum. If one assumes that f is proper and X is completely paracompact, the compactness of X may be discarded. Under these hypotheses, the assumption that A satisfies the homological condition above is sufficient to guarantee that Y is an orientable n-cm.

We apply Theorem 1 to give a generalized version of R. L. Wilder's monotone mapping theorem [5].

In what follows, X will denote a connected, orientable n-cm, and γ will denote the fundamental class of X ($\gamma \in H_n^c(X)$). If A is a continuum in X, then $c\colon X \to X/A$ is the canonical identification, and c(A) is represented by $*$. If $S \subset X/A$, then $c^{-1}(S) = S^*$. A proper, compact, connected subset A of X is called a *divisor* of X if the homomorphism

Received January 12, 1970.
This paper is part of the author's dissertation, prepared under the supervision of Professor K. W. Kwun at Michigan State University.

Michigan Math. J. 17 (1970).