THE HOMOTOPY GROUPS OF BPL AND PL/O. III

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1. INTRODUCTION

Let Γ_{k-1} denote the group of oriented differentiable structures on the (k - 1)-sphere. M. Kervaire and J. Milnor [8] have constructed an exact sequence

1.0 $$0 \to bP_k \to \Gamma_{k-1} \to \pi^s_{k-1}/\mathrm{im}(J),$$

where $bP_k \subseteq \Gamma_{k-1}$ is the subgroup of exotic spheres that bound π-manifolds, π^s_{k-1} is the stable (k - 1)-stem, and J is the classical J-homomorphism. Further, it is known that

$$bP_k = \begin{cases} \mathbb{Z}_{\theta_k} & \text{if } k \equiv 0 \pmod 4, \\ \mathbb{Z}_2 & \text{if } k \equiv 2 \pmod 4 \text{ and } k \neq 2^j - 2, \\ 0 & \text{if } k \equiv 1 \text{ or } k \equiv 3 \pmod 4, \end{cases}$$

where θ_k is a large integer, and that $\Gamma_{k-1} \to \pi^s_{k-1}/\mathrm{im}(J)$ is surjective if $k - 1 \neq 2^j - 2$ [8], [3].

In [5] and [6], we showed that the exact sequence 1.0 splits if $k \equiv 0$, 2, or 4 (mod 8). That is,

$$\Gamma_{4m-1} \simeq \mathbb{Z}_{\theta_{4m}} \oplus (\pi^s_{4m-1}/\mathrm{im}(J)) \quad \text{and} \quad \Gamma_{8m+1} \simeq \mathbb{Z}_2 \oplus (\pi^s_{8m+1}/\mathrm{im}(J)),$$

for all m. In this note, we outline a proof of the splitting of the sequence 1.0 for $k \equiv 2 \pmod 4$ but $k \neq 2^j - 2$.

THEOREM 1.1. *There exists an isomorphism*

$$\Gamma_{4m+1} \simeq \mathbb{Z}_2 \oplus (\pi^s_{4m+1}/\mathrm{im}(J))$$

if $4m + 2 \neq 2^j - 2$.

Theorem 1.1 includes the dimensions $k \equiv 2 \pmod 8$ dealt with in [6]. The proof given here is perhaps more elementary.

In the dimensions $2^j - 2$, it is known that $bP_{2^j-2} = 0$ if the element $(h_{j-1})^2$ survives to E_∞ in the Adams spectral sequence [3]. If $bP_{2^j-2} = 0$, there is no splitting problem in the exact sequence 1.0. M. Mahowald has shown that $(h_{j-1})^2$ does, in fact, survive to E_∞, if $j \leq 6$.

There exists an isomorphism $\Gamma_{k-1} \simeq \pi_{k-1}(PL/O)$ due to M. Hirsch and B. Mazur [7]. From the fibration $PL/O \to BO \to BPL$, it is clear that $\pi_k(BPL) \simeq \pi_{k-1}(PL/O)$ for $k \equiv 6 \pmod 8$, since $\pi_k(BO) = \pi_{k-1}(BO) = 0$. In [5] and

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