# PERIODIC SOLUTIONS OF THE TRICOMI PROBLEM

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## INTRODUCTION

Consider the system of differential equations

$$(0.1)\qquad \begin{cases} u_x(x, y) = F(x, y, u, v), \\ v_y(x, y) = G(x, y, u, v) \end{cases}$$

and the boundary conditions

$$(0.2)\qquad u(0, y) = \tau(y), \qquad v(x, 0) = \sigma(x).$$

Here $F(x, y, u, v)$ and $G(x, y, u, v)$ are continuous vector functions (possibly of different dimensions) defined in a region

$$|x| \leq a_1 \leq \infty, \qquad |y| \leq a_2 \leq \infty, \qquad |u| \leq u_1, \qquad |v| < v_1,$$

while $\sigma(x)$ and $\tau(y)$ are prescribed continuous vector functions defined for $|x| \leq a_1 \leq \infty$ and $|y| \leq a_2 \leq \infty$. *Tricomi's problem* (see [10]) is to find two vector functions $u(x, y)$ and $v(x, y)$ that, together with $u_x$ and $u_y$, are continuous in $|x| \leq a_1$, $|y| \leq a_2$, and that satisfy the system (0.1) and the boundary conditions (0.2) on $|x| < a_1$, $|y| < a_2$. We emphasize that a solution of (0.1) - (0.2) does not generally have a continuous second-order mixed partial derivative, so that the system (0.1) - (0.2) cannot generally be reduced to a system of the form $w_{xy} = H(x, y, w, w_x, w_y)$.

Under some regularity conditions on F and G (see F. Tricomi [8], G. Villari [10], G. Santagati [7]), there exists at least one solution of problem (0.1) - (0.2); under other more restrictive conditions, there exists a unique solution of (0.1) - (0.2). In this paper we investigate the existence of periodic solutions for the Tricomi problem. We use a method of L. Cesari [5], which consists in treating first a slightly modified (relaxed) problem. In particular, L. Cesari used the method in [2], [3], and [4] to obtain the periodic solutions of the Darboux problem

$$u_{xy} = f(x, y, u, u_x, u_y), \qquad u(x, 0) = \nu(x), \qquad u(0, y) = \mu(y).$$

We remark that the problem (0.1) - (0.2) is equivalent to the problem of finding continuous solutions of the system

$$u(x, y) = \tau(y) + \int_0^x F[\xi, y, u(\xi, y), v(\xi, y)]d\xi,$$

$$v(x, y) = \sigma(x) + \int_0^y G[x, \eta, u(x, \eta), v(x, \eta)]d\eta.$$

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