THE BROUWER PROPERTY AND INVERT SETS

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1. INTRODUCTION

A topological space X is said to have the *Brouwer property* if homeomorphic images of open subsets of X are also open subsets of X (see G. T. Whyburn [9], [10], and [11]). Thus, euclidean spaces and manifolds have the Brouwer property, whereas manifolds with nonempty boundary do not. For $n < 3$, E. Duda [3] showed that an n-complex has the Brouwer property if and only if it is an n-manifold.

Invertible spaces were introduced by P. H. Doyle and J. G. Hocking [2]; a point p of a topological space X is an *invert point* if for each open neighborhood U of p there exists a homeomorphism h of X onto itself such that $h(X - U) \subseteq U$. If h is isotopic to id_X, then p is a *continuous invert point*. The collection of all invert points is the *invert set*, denoted by I(X). The *continuous invert set* CI(X) is defined similarly. Doyle [1] investigated invert sets in complexes, and he showed that for each complex K, the set I(K) is the empty set, a point, or a simplicial sphere. Hocking proved that if $I(K) = S^k$ $(0 \leq k \leq n)$, then the n-complex K is a multiple suspension. An n-complex K with a single-point invert set was characterized by Doyle [1] and by V. M. Klassen [7] for $n = 1$ and 2. In this paper, we discuss n-complexes having the Brouwer property, and we focus our attention on the case where $n = 3$ and I(K) is a single point.

2. A CHARACTERIZATION OF THE 3-SPHERE

It is easily seen that if K is an n-complex with the Brouwer property and $I(K) = \{p\}$, then Lk(p) has the Brouwer property. Also, a complex L has the Brouwer property if its suspension $\mathscr{S}(L)$ has the Brouwer property.

THEOREM 1. *Let* K *be a* 3-*complex with the Brouwer property. Then* $\dim\{I(K)\} \geq 1$ *if and only if* $K = S^3$.

Proof. If $K = S^3$, then $I(K) = S^3$. On the other hand, if $\dim\{I(K)\} \geq 1$, we can write $K = \mathscr{S}(L)$, where L is a 2-complex with the Brouwer property. By Duda's result, L is a 2-manifold. Moreover, there exist x and y in L such that $\{x, y\} \subseteq L \cap I(K)$. But since L is a manifold, $L \subseteq I(K)$. Thus $K = \mathscr{S}(L) \subseteq I(K)$. Consequently, $K = I(K)$, and by [2], $K = S^3$.

3. ORBITS

Let K be a 3-complex, with $I(K) = S^0$, and possessing the Brouwer property. Then $K = \mathscr{S}(L)$, where L is a 2-manifold M^2. It is possible that M^2 is a disjoint union of m 2-manifolds $(m \geq 1)$. From such a complex we can obtain another with a single-point invert set, by identifying the two suspension points of $\mathscr{S}(L)$ (see Theorem 3).

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