INTRODUCTION

If b is an integer exceeding 1, then each positive integer n is uniquely expressible as $n = \sum_{i=0}^{\infty} \alpha(n, i)b^i$, where each $\alpha(n, i)$ is a nonnegative integer less than b. Numerous analogies exist between this representation and the representation $n = \prod_{i=1}^{\infty} p_i^{\beta(n,i)}$ given by the fundamental theorem of arithmetic. For instance, if we put $m \prec n$ if $\alpha(m, i) \leq \alpha(n, i)$ for each i, it is clear that "$\prec$" is analogous to "divides."

We say that a sequence C is b-*complete* provided that the relations $x \in C$ and $x \prec y$ imply that $y \in C$. A b-complete sequence is analogous to a sequence of integers formed by taking all multiples of the elements of a fixed sequence (sequences of this form have been studied extensively in density theory).

H. Davenport and P. Erdös [5] proved that a sequence formed by taking all multiples of the elements of a fixed sequence possesses a logarithmic density. They also proved that a sequence with positive upper logarithmic density contains a division chain, that is, an infinite subsequence in which each member divides its successor.

In giving examples of situations in which his "magnification theorem" holds, E. M. Paul [9], [10] provided another proof of the Davenport-Erdös theorems. It is implicit in the work in his dissertation [9] that a b-complete sequence possesses natural density, and that a sequence with positive upper natural density contains a chain $d_1 \prec d_2 \prec \cdots$.

In [1] we introduced a method that allowed us to extend the results of Davenport and Erdös; and in this paper we adapt this method to the study of digits. In particular, we examine the structure of chains that exist under various density conditions.

Another well-investigated type of sequence is the primitive sequence (here, no member divides another).

The following two fundamental results are due to P. Erdös [6] and F. Behrend [3], respectively: if $a_1, a_2, \cdots$ is a primitive sequence, then

1) $\sum_i (a_i \log a_i)^{-1} \leq M$, where M is an absolute constant,

2) $\sum_{a_i \leq x} 1/a_i \leq K(\log x)/\sqrt{\log \log x}$.

Our analogous results are that if $a_1, a_2, \cdots$ is a b-primitive sequence, that is, if $a_i \not\prec a_j$ whenever $i \neq j$, then

1) $\sum_i 1/a_i \leq b$,

2) $\sum_{a_i \leq x} 1 \leq Kx/\sqrt{\log x}$.

Received July 13, 1967.
This research was supported in part by Army Research Office - Durham, Grant ARO-D-31-124-G892.