LIGHT OPEN MAPPINGS ON A TORUS WITH A DISK REMOVED

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1. INTRODUCTION

Suppose δ is a continuous mapping of a Jordan curve J into Euclidean 2-space R^2. We shall consider J as the boundary of T, a 2-dimensional torus with a disk removed, and also as the boundary of a 2-cell D, and we shall study the relation between the case where δ has a light, open, continuous extension to T and the case where δ has a similar extension to D.

Generally, definitions and notation not given in the paper will be as in [3]. All light open mappings will be assumed to be sense-preserving, unless it is otherwise specified.

2. MAIN RESULTS

Definition 1. Suppose J is a Jordan curve on a 2-dimensional torus that bounds a disk; let T be the other component of the complement of J. Suppose δ is a continuous mapping of J into R^2. We say that δ is a t-*boundary* if there exists a properly interior mapping f: $\overline{T} \to R^2$ such that $f \mid J = \delta$. (We use the term *properly interior* in the sense of [7], not [3].)

We shall consider J as embedded in R^2 and oriented as in Figure 1.

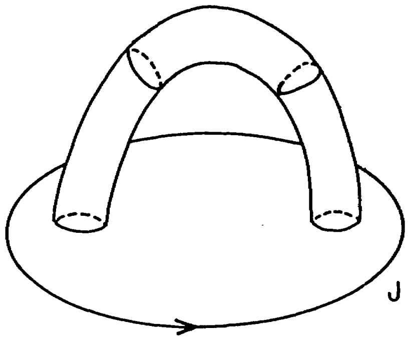

Figure 1.

Definition 2. Suppose I = [a, b] is a closed interval of real numbers, A is some closed arc, and δ: A $\to R^2$. We extend the definition of normality as in [6, p. 1084] and say that δ is *topologically normal* (briefly, t-*normal*) if there exist homeomorphisms h: I $\to$ A and k: $R^2 \to R^2$ such that $k \circ \delta \circ h$ is normal in the sense of [6]. Also, if M is an oriented Jordan curve and δ maps M into R^2, then δ is t-*normal* if there exists a mapping ψ: I $\to$ M such that $\psi(a) = \psi(b)$, ψ is one-to-one on (a, b), $\psi(x) \neq \psi(a)$ for $x \in (a, b)$, and $\delta \circ \psi$ is t-normal as defined in the previous sentence. Let δ map the closed arc A_1 into R^2; let η map the closed arc A_2 into R^2. We say that δ and η *intersect* t-*normally* if there exist homeomorphisms h_1: I $\to A_1$, h_2: I $\to A_2$, and k: $R^2 \to R^2$ such that $k \circ \delta \circ h_1$ and $k \circ \eta \circ h_2$ intersect normally in the sense of [3, p. 50].

LEMMA 1. *Let* U *be an open connected subset of a metrizable 2-dimensional manifold, and suppose* f: U $\to R^2$ *is light and open. For any two points* p *and* q *in* U, *there exists an arc* A *in* U *with end points* p *and* q *such that* $f \mid A$ *is* t-*normal. Also, if* A_1 *is any arc and* g: $A_1 \to R^2$ *is* t-*normal, then* A *can be chosen so that* $f \mid A$ *and* $g \mid A_1$ *intersect* t-*normally. Finally, if* U *is bounded by a finite number of Jordan curves and* f *is a local homeomorphism at points of* $\overline{U}$ - U, *then the*

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