SYMMETRIC SPACES AND PRODUCTS OF SPHERES

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1. INTRODUCTION

In this paper we extend to certain symmetric spaces a result of J.-P. Serre [8] on the comparison of Lie groups and products of spheres. Throughout the paper, G will denote a compact, connected, simply connected Lie group, σ: G $\to$ G an automorpism of period 2, and K the identity component of the fixed point set of σ. We shall assume that K is totally nonhomologous to zero in G with real coefficients, that is, that the inclusion $K \subset G$ induces an epimorphism in real cohomology. It is known [4] that under these hypotheses G/K has the same real cohomology as a product

$$X = S^{n_1} \times \cdots \times S^{n_\ell} \qquad (n_1 \leq n_2 \leq \cdots \leq n_\ell;\ n_i \text{ odd}).$$

A prime p is *regular* for G/K if there exists f: X $\to$ G/K such that

$$f^*: H^*(G/K; Z_p) \to H^*(X; Z_p)$$

is an isomorphism.

THEOREM 1. *If* p *is an odd prime,* $p \geq (n_\ell + 1)/2$, *and* G/K *has no* p-*torsion, then* p *is regular for* G/K.

This theorem, the proof of which is given in Section 2, extends Proposition 6 in Chapter V of Serre's paper [8], which under similar hypotheses gives the above conclusion for a Lie group. The converse of Serre's result on the regularity of primes for a Lie group was proved by Serre [8] for classical groups, and by the author [6] for the exceptional groups. Here we give a proof, using the classification of irreducible symmetric spaces, of a partial converse of Theorem 1 in the case where G is classical:

THEOREM 2. *If* G *is a classical group and* G/K *is an irreducible symmetric space different from a sphere, then each prime* $p < (n_\ell + 1)/2$ *is irregular for* G/K.

The irreducible symmetric spaces to which Theorem 1 applies are

(i) $(K \times K)/K$ (K a simple Lie group),

(ii) $SU(2n+1)/SO(2n+1)$,

(iii) $SU(2n)/Sp(n)$,

(iv) $Spin(2n)/Spin(2n-1)$,

(v) E_6/F_4.

In Section 4 we show that the only obstacle to the elimination from Theorem 2 of the hypothesis that G is classical is a proof that the prime 7 is irregular for E_6/F_4.

Received October 20, 1966.
This research was supported in part by a State University of New York Faculty Research Fellowship.