# RIEMANNIAN MANIFOLDS OF CONSTANT NULLITY

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## 1. INTRODUCTION

Let m be a point of a ($C^\infty$) Riemannian manifold M, and let $M_m$ denote the tangent space to M at m. A vector $z \in M_m$ is called a *nullity vector* if $R_{xy}z = 0$ for all x, $y \in M_m$, where $R_{xy}$ denotes the curvature transformation associated with the vectors x and y. The *nullity* $\mu(m)$ is the dimension of the space of nullity vectors at m. The purpose of this paper is to prove the following theorem concerning Riemannian manifolds with constant positive nullity.

THEOREM (*). *Let* $M^n$ *be a complete, connected, and simply connected* $C^\infty$ *Riemannian manifold of constant positive nullity* $\mu \leq n - 3$, *and suppose that one of the following conditions is satisfied:*

(1*) $n - \mu$ *is odd, and the sectional curvatures of all planes orthogonal to the spaces of nullity vectors are nonzero;*

(2*) *the restriction of the curvature tensor to the space of bivectors generated by vectors orthogonal to the space of nullity vectors at each* $m \in M$ *is a positive or negative definite bilinear form on this space.*

*Then* $M^n$ *is a direct metric product,* $M^n = N^\mu \times C^{n-\mu}$, *where* $N^\mu$ *is complete and flat, and* $C^{n-\mu}$ *is complete.*

Nullity was defined by Chern and Kuiper [2]. Theorem (*) is a $C^\infty$ intrinsic-manifold analogue of a theorem due to Hartman [4], who assumed the existence of an immersion.

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## 2. PRELIMINARIES: THE NULLITY VARIETIES AND CONULLITY OPERATORS

Throughout this paper, M will denote an n-dimensional differentiable Riemannian manifold (of class $C^\infty$). The frame bundle, solder form, connexion form, and curvature form of $\underline{M}$ will be denoted respectively by $\overline{F}(M)$, $\overline{\theta}$, $\overline{\omega}$, and $\overline{\Omega}$ [1]. The natural projection of $\overline{F}(M)$ onto M will be denoted by $\overline{\pi}$. We shall use the index convention in which a repeated index means summation through all possible values of the index. The subspace $N_m$ of $M_m$ generated by the nullity vectors at m is called the *nullity space* at m. A *conullity vector* is a vector orthogonal to $N_m$, and the subspace $C_m$ of conullity vectors at m is called the *conullity space* at m.

In this section we shall suppose only that the nullity is positive and constant in some open set of M. With this assumption, we can find (locally) the flat factor of the

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