PIECEWISE LINEAR UNKNOTTING OF $S^p \times S^q$ IN S^{p+q+1}

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INTRODUCTION

Denote by S^n the unit n-sphere in euclidean $(n+1)$-space, and by D^n the unit n-ball in euclidean n-space.

J. W. Alexander [2] proved that if $S^1 \times S^1$ is piecewise linearly embedded in S^3, then the closure of one of the components of $S^3 - S^1 \times S^1$ is homeomorphic to $S^1 \times D^2$. Alexander's method is based on the study of the intersections of a plane with $S^1 \times S^1$ as the plane moves through euclidean 3-space.

A. Kosinski [6] generalized this result by showing that every product $S^p \times S^q$ differentiably embedded in S^{p+q+1} can be unknotted differentiably in S^{p+q+1}, provided $p > q > 1$, $p + q > 5$, and p is odd in case $q = 2$. For this, he used Smale theory to show that one of the components of $S^{p+q+1} - S^p \times S^q$ is diffeomorphic to $S^q \times D^{p+1}$. Then, using the fact that S^q unknots differentiably in S^{p+q+1} under the above assumptions on p and q, he was able to unknot $S^p \times S^q$ in S^{p+q+1}. He asked whether the same result is true in the PL (piecewise linear) category. Our purpose is to answer this question and to drop some of the condition on p and q. (This result has been proved independently by C. T. C. Wall.)

Reformulating Alexander's theorem, we can say that if $S^1 \times S^1$ is PL embedded in S^3, then the closure of one of the components is a regular neighborhood of some 1-sphere embedded in S^3. We generalize this reformulation, by proving that if there is a locally unknotted PL embedding of $S^p \times S^q$ in S^{p+q+1}, where $p \geq q > 1$ and $p + q > 4$, then the closure of one of the components of $S^{p+q+1} - S^p \times S^q$ is a regular neighborhood of a p-sphere embedded in S^{p+q+1}. Using Zeeman's unknotting theorem and Whitehead's regular neighborhood theorem [12], we can then show that $S^p \times S^q$ unknots in S^{p+q+1}. We need the restriction that the embedding be locally unknotted, since the Schoenflies conjecture has not been proved in the piecewise linear category for dimension greater than 3; thus there is an essential difficulty in local unknotting.

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1. THE PIECEWISE LINEAR CATEGORY

Throughout this paper, all *simplicial complexes* shall be finite simplicial complexes. Sometimes we shall revert to polyhedra, in order to avoid excessive subdivision. By a *polyhedron* we mean the space underlying a finite simplicial complex; and by a *subpolyhedron of a simplicial complex*, we mean the subspace underlying a subcomplex of some rectilinear subdivision.

1.1. *Definition.* If K and L are simplicial complexes, then a map $f: K \to L$ is said to be *piecewise linear* if there exist rectilinear subdivisions K' and L' of K

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