TWO HOMOMORPHIC BUT NONISOMORPHIC MINIMAL SETS

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Let (X, T, π) be a transformation group with compact Hausdorff phase space [5]. We say that (X, T, π) is a minimal set if and only if for every point x in X the orbit closure $\overline{O(x, T)} = \overline{\{xt: t \in T\}}$ is the space X. The classification of minimal sets is one of the important problems in topological dynamics. Although significant progress has recently been made [4], the problem is far from solved. In a forthcoming paper [2], J. Auslander classifies the minimal sets by means of homomorphisms. A homomorphism $\theta: (X, T) \to (Y, T)$ is a continuous map from X into Y such that $x\theta t = xt\theta$ for all $t \in T$, $x \in X$. In regard to such classifications, the following question naturally arises: *If* (X, T) *and* (Y, T) *are compact minimal sets having homomorphisms* $\theta: (X, T) \to (Y, T)$ *and* $\phi: (Y, T) \to (X, T)$, *does there exist an isomorphism from* (X, T) *onto* (Y, T)?

In this note, we shall show by an example that the answer to this question is negative. Our minimal sets are based on minimal sets given by R. Ellis (see [4, Example 4] or [1, p. 613]); we shall describe these first.

1. Let Y denote the additive group of real numbers modulo 1, let Y_1 and Y_2 be two disjoint copies of Y, and let $X = Y_1 \cup Y_2$. For each $y \in Y$, corresponding points in Y_1 and Y_2 will be written as $(y, 1)$ and $(y, 2)$, respectively. Topologize X by specifying an open-closed neighborhood system for each point. If $\varepsilon > 0$, let

$$N_\varepsilon(y, 1) = \{(y+t, 1): 0 \leq t \leq \varepsilon\} \cup \{(y+t, 2): 0 < t < \varepsilon\}$$

be an open-closed neighborhood of $(y, 1) \in Y_1$, and let

$$N_\varepsilon(y, 2) = \{(y+t, 2): 0 \geq t \geq -\varepsilon\} \cup \{(y+t, 1): 0 > t > -\varepsilon\}$$

be an open-closed neighborhood of $(y, 2) \in Y_2$. For $i = 1, 2$, let $\tau: X \to X$ be defined by the formula $(y, i)\tau = (y + \alpha, i)$, where α is a real number. Then τ is a self-homomorphism of X.

1.1 LEMMA. *Let* T *denote the group generated by* τ *and topologized by the discrete topology. Then*

(X, T) *is a transformation group, and*

X *is a compact, separable Hausdorff space satisfying the first countability axiom.*

1.2 *Definition.* Let us call the real number α associated with τ the *rotation constant* of τ (or of T). Then (X, T) is a minimal set when the rotation constant is an irrational number. In this case, two points u and v in Y are proximal [4] if and only if $u = (y, i)$ and $v = (y, j)$ for some $y \in Y$.

Now we shall proceed to construct our minimal sets.

2. Let α, β, and γ be three real numbers such that α, β, γ, and 1 are rationally independent. Let (X, T) be the Ellis minimal set with rotation constant α, and let

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