TWO PROBLEMS IN THE THEORY OF GENERALIZED MANIFOLDS

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Dedicated to Professor R. L. Wilder on the occasion of his seventieth birthday.

At the R. L. Wilder Conference in Topology at the University of Michigan, held in honor and appreciation of R. L. Wilder's contributions to topology, I discussed three problems in the theory of generalized manifolds that have interested me. This is a report on two of these problems.

1. DIMENSION OF GENERALIZED MANIFOLDS

We adopt the conventions of [1] and call a locally orientable generalized n-manifold M over a principal ideal domain L a *cohomology* n-*manifold over* L (M is an n-cm over L). This differs slightly from the terminology of Wilder [12] in that no assumption on the covering dimension of M is made. It is known that the cohomological dimension of M over L, to be denoted by $\dim_L M$, is exactly n. Whether or not the covering dimension of M is finite is still unknown. Several other interesting questions concerning the covering dimension of generalized manifolds have been stated by Wilder in [12, p. 382]. The recent reprinting of [12] (1963) contains a discussion of the present status of these questions.

PROBLEM 1. *Let* M *be an* n-cm *over* L. *Is* $\dim_{Z_p} M = \dim_Q M = \dim_L M$, *for all primes* p?

If Z_p or Q are L-modules, then M is also an n-cm over the respective Z_p or Q. Also, if $n \leq 2$ and M is separable metric, then M is locally Euclidean by a theorem of Wilder [12, pp. 275-280]. Hence, the answer to Problem 1 is partially known. However, the question is unanswered in the following special situation.

(i) *Let* M *be an* n_p-cm *over* Z_p *for each prime* p *and the rational field* (p = 0). *Is* n_p *independent of* p? *Is* M *a* cm *over* Z?

It may be possible to answer affirmatively the second part of (i) with the added assumption that M is clc over Z (see [8, p. 1375]).

An affirmative answer to either question in (i) would imply that every compact effective group of homeomorphism of a manifold (or separable metric cohomology manifold) is a Lie group (see [7] for more details). This problem, of course, is the generalized Fifth Problem of Hilbert. The answer is still unknown. We can indicate a feeling for the connections between these two problems by considering a special case. Assume that the p-adic group A_p operates freely on an orientable n-cm M over Z. Consider the space $(M \times \Sigma_p)/A_p$, where the action of A_p is the diagonal action and Σ_p is the p-adic solenoid. This space can be fibered over the circle with fiber M. Hence $(M \times \Sigma_p)/A_p = M'$ is an $(n+1)$-cm over Z. The p-adic solenoid now operates freely on M', so that M'/Σ_p is homeomorphic to M/A_p, the space

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