EXAMPLES OF GENERALIZED-MANIFOLD APPROACHES TO TOPOLOGICAL MANIFOLDS

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Dedicated to R. L. Wilder on his seventieth birthday.

INTRODUCTION

The theory of generalized manifolds and the study of topological manifolds have no doubt influenced each other in methods and motivations. Of course, the theory of topological manifolds has its own approaches, and the extent to which we can apply the theory of generalized manifolds is necessarily limited. At times, however, the latter theory may offer a better picture of the problem involved, and it may even play an essential role. I hope to illustrate this point with a few simple examples.

1. SEPARABLE 1- AND 2-gms

One of the most useful aspects of gms in connection with topological manifolds is the fact that 1- and 2-gms that are separable are actual manifolds [8, Chapter 9]. This is not the case for dimensions greater than 2. In fact, the monotone mapping theorem [9], [10] of Wilder opened a convenient way of constructing gms that are not manifolds, for dimensions exceeding 2. In order to show that a given space be a 2-manifold, it therefore suffices to prove that it is a 2-gm. To give an example of this approach, we recall the following famous theorem of R. L. Moore.

If G *is an upper-semicontinuous decomposition of the plane into continua that do not separate the plane, then the decomposition space is homeomorphic to the plane.*

This theorem is an immediate corollary of the monotone mapping theorem of Wilder that I have already mentioned. I point out that despite appearances to the contrary, theorems on generalized manifolds sometimes have significant implications concerning manifolds.

2. HANDLING OF BOUNDARIES

By this time, it is well known that a cartesian factor of a manifold need not be a manifold (see [1]). Suppose A and B are spaces, and suppose one wishes to show that $A \times B$ is a manifold with nonempty boundary. It is often necessary to consider separately the candidate for $\mathrm{Bd}(A \times B)$ and that for $\mathrm{Int}(A \times B)$. This means that one has to analyze $\mathrm{Int}(A \times B)$ in terms of A and B. Since A and B need not be manifolds, this may pose a difficulty. The most natural way to take care of this is to regard $A \times B$ as a gm and to use the formula $\mathrm{Bd}(A \times B) = \mathrm{Bd}\, A \times B \cup A \times \mathrm{Bd}\, B$. Here Bd is taken in the sense of gms, and this is possible because by the factorization theorem [6] of Raymond A and B are gms.

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