MAXIMALLY ALMOST PERIODIC AND UNIVERSAL EQUICONTINUOUS MINIMAL SETS

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Let T be a topological group. A transformation group (X, T) with compact phase space X is called a *universal equicontinuous minimal set for* T if (X, T) is equicontinuous minimal and if each equicontinuous minimal transformation group (Y, T) with compact phase is a homomorphic image of (X, T). A metatheorem concerning universal "admissible" minimal sets [2, Theorem 2] guarantees the isomorphically unique existence of such a universal object for T.

Chu [3] has defined in a different way the notion of a universal almost periodic minimal set for T, called here a *maximally almost periodic minimal set for* T (to avoid ambiguity caused by the equivalence of equicontinuity and almost periodicity for a transformation group with compact phase space).

We discuss below the relations to each other and to group compactifications of T of these two types of objects. If a transformation group (X, T) is a universal equicontinuous minimal set for T, then it is a maximally almost periodic minimal set for T; we show that the converse holds if T is compact and (X, T) is effective. We prove the existence of many maximally almost periodic minimal sets for noncompact generative T that are not universal equicontinuous minimal sets for T. As general references on the category of minimal transformation groups, see [4] and [5].

All compact and locally compact spaces considered are assumed to be Hausdorff spaces.

For a topological space X, $C^*(X)$ denotes the Banach algebra of bounded continuous real-valued functions on X with the uniform norm, and ϕ^* denotes the canonical map of $C^*(Y)$ into $C^*(X)$ induced by a continuous map ϕ of X into a space Y.

Let T_d denote the group underlying T provided with its discrete topology. Let $(C^*(T), T_d, \lambda)$ be the transformation group of isometric automorphisms of $C^*(T)$ given by

$$s((f, t)\lambda) = (ts)f \qquad (t, s \in T;\ f \in C^*(T)).$$

Let $A^*(T)$ be the T_d-invariant subalgebra of $C^*(T)$ consisting of those functions in $C^*(T)$ that are (left) almost periodic, that is, whose orbits under λ are relatively compact. We denote by λ again the restriction of λ to $A^*(T) \times T_d$.

We call a couple (X, ϕ), where X is a compact space and ϕ is a continuous map of T into X, an *almost periodic compactification of* T if $T\phi$ is dense in X and $C^*(X)\phi^* = A^*(T)$, so that ϕ^* is an isometric isomorphism of $C^*(X)$ with $A^*(T)$. A *maximally almost periodic minimal set for* T is then an equicontinuous minimal transformation group (X, T) with compact phase space X such that (X, ϕ) is an almost periodic compactification of T for some ϕ. (Chu's definition of a universal almost periodic minimal set imposes an additional condition that is superfluous, in view of Corollary 2.) Clearly, any two maximally almost periodic minimal sets for

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