UNIVERSAL $\mathscr{P}$-LIKE COMPACTA

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1. INTRODUCTION

A *compactum* is a compact, metrizable space. A *continuum* is a connected compactum. By a *polyhedron* we mean a finitely triangulable space.

If $\mathscr{C}$ is a class of spaces, a *universal* member of $\mathscr{C}$ is a member of $\mathscr{C}$ in which every member of $\mathscr{C}$ can be imbedded. K. Menger [11] described an n-dimensional continuum which he conjectured (and proved in the case $n = 1$) to be a universal n-dimensional compactum. G. Nöbeling [12] produced a different space, which he showed to be a universal n-dimensional, separable, metrizable space. S. Lefschetz [8] (independently of [12]) verified Menger's conjecture. See Hurewicz and Wallman [7, p. 64] for a treatment of Nöbeling's theorem. R. M. Schori [13], [14], has shown that there exist universal snake-like continua (see R. H. Bing [1]).

We present a single, rather general theorem (Theorem 1) that implies (see Theorem 2) both Schori's result and the existence of universal n-dimensional compacta. The method of proof, involving inverse limit systems, is an extension of Schori's method. Lefschetz's proof of Menger's conjecture used a version of polyhedral inverse limit expansions. The main feature of the present approach is the additional use of polyhedral inverse limit systems to *define* the required universal spaces.

The framework needed for our theorems is the theory of $\mathscr{P}$-like compacta, where $\mathscr{P}$ is a class of polyhedra. See Mardešić and Segal [9]. If α is an open cover of the compactum X, a map f of X onto a compactum Y is called an *α-map* provided that for each y in Y, $f^{-1}(y)$ is contained in some member of α. Let $\mathscr{P}$ be a class of polyhedra. Following [9], we say a compactum X is *$\mathscr{P}$-like* if for each open cover α of X there exists an α-map of X onto some member of $\mathscr{P}$.

We are concerned with the following question: For which classes $\mathscr{P}$ is there a universal $\mathscr{P}$-like compactum? Theorems 1 and 2 are positive results; Theorems 3 and 4 are negative. Part of the results were announced in [10].

2. STATEMENT OF THEOREMS

Definition 1. The class $\mathscr{P}$ of polyhedra is called *amalgamable* if for each finite sequence $(P_1, \cdots, P_n)$ of members of $\mathscr{P}$ and maps $\phi_i\colon P_i \to Q$ $(1 \leq i \leq n)$, where $Q \in \mathscr{P}$, there exists a member P of $\mathscr{P}$ with imbeddings $\mu_i\colon P_i \to P$ and a map ϕ of P onto Q such that $\phi_i = \phi\mu_i$ for each i. We call $(P, \phi, \mu_1, \cdots, \mu_n)$ an *amalgamation* of $(\phi_1, \cdots, \phi_n)$.

THEOREM 1. *If $\mathscr{P}$ is an amalgamable class of polyhedra, then there exists a universal $\mathscr{P}$-like compactum.*

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