NONLINEAR PERTURBATION OF A LINEAR SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS

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Given two systems of ordinary differential equations,

$$\dot{x} = A(t)x + f(t, x), \tag{1}$$

$$\left(\cdot = \frac{d}{dt}\right)$$

$$\dot{x} = A(t)y \tag{2}$$

and a fundamental matrix $Y(t)$ for (2), we pose the following problems:

(i) If $x(t)$ is a solution of (1), does there exist a constant $n \times 1$ matrix b such that

$$x(t) = Y(t)[b + o(1)] \text{ as } t \to \infty\,? \tag{3}$$

(ii) If b is a constant $n \times 1$ matrix, does there exist a solution $x(t)$ of (1) such that (3) holds?

In Theorem 1 we generalize the results of Z. Szmydt [11, Theorems 1 and 2], and we give a positive answer to Problem (i). Theorem 1 is also a generalization of a result of R. Bellman [2], who studied the case in which $f(t, x)$ is linear.

A positive answer to Problem (ii) is given in Theorem 2. This theorem depends on the Lemma stated below, which is a very special case of one of the author's earlier results [8, Theorem 1].

A special case of Theorems 1 and 2 is considered in the Corollary following Theorem 2.

In Theorem 3 we give a generalization of a result of W. Trench [12]. See also [1, Theorem 2], [5] and [9]. Our Theorem 3 is a positive answer to Problem (ii) for the case in which second-order systems are considered. Trench deals with second-order scalar equations under linear perturbations. We deal with second-order systems under perturbations not necessarily linear. The proof of Theorem 3 depends on the Corollary mentioned above.

Results related to problems (i) and (ii) may be found in [3], [6], [7], and [10]. Other references can be found in the book by L. Cesari [4].

We denote by $\|z\| = \Sigma_j |z_j|$ the norm of any $n \times 1$ matrix $z = \mathrm{col}(z_1, \cdots, z_n)$ and by $\|Z\| = \Sigma_{i,j} |Z_j^i|$ the norm of any $n \times n$ matrix $Z = (Z_j^i)$. Our results are dependent upon the following hypothesis.

HYPOTHESIS H. *For every positive constant* M *there exists a nonnegative function* $h_M(t)$ *such that if* Y *is a fundamental matrix for* (2), *then*

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