JORDAN DOMAINS AND ABSOLUTE CONVERGENCE OF POWER SERIES

G. Piranian

Let J denote a Jordan curve that separates the origin from the point at infinity, let J^{-1} denote the image of J under reflection in the unit circle, and let the functions

$$f(z) = \sum_1^\infty a_n z^n, \tag{1}$$

$$g(z) = b_{-1} z^{-1} + \sum_0^\infty b_n z^n, \tag{2}$$

$$h(z) = \sum_1^\infty c_n z^n \tag{3}$$

provide conformal schlicht mappings of the unit disk D onto the interior of J, the exterior of J, and the interior of J^{-1}, respectively,

We know little about the relation between geometric properties of J and the absolute convergence of the series Σa_n, Σb_n, Σc_n, beyond the facts that rectifiability of J implies the absolute convergence of all three series and that (for example) absolute convergence of Σa_n implies uniformly rectifiable accessibility of J from the interior. (The boundary of a domain B is *uniformly rectifiably accessible* if to each point p in B there corresponds a finite constant $S = S(p)$ such that each boundary point of B can be joined to p by a path that lies in B and has length at most S. In the case of the function g, the relation between absolute convergence and uniformly rectifiable accessibility follows from the inequalities

$$\int_{1/2}^1 |g'(re^{i\theta})|\, dr \leq \int_{1/2}^1 |b_{-1}|\, r^{-2}\, dr + \int_{1/2}^1 \sum_1^\infty n|b_n|\, r^{n-1} dr$$

$$\leq |b_{-1}| + \sum_1^\infty |b_n|$$

and the fact that g maps the circle $|z| = 1/2$ onto a rectifiable curve.)

In a recent conversation, Ch. Pommerenke asked whether absolute convergence of Σa_n implies absolute convergence of Σb_n.

THEOREM. *There exists a Jordan curve* J *such that the functions* (1), (2), *and* (3) *satisfy the conditions*

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