TRANSFORMATION GROUPS WITH ORBITS OF UNIFORM DIMENSION

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1. INTRODUCTION

The purpose of this paper is to prove a local analogue of a result of Borel and Conner. The present theorem states that if G is a compact Lie group acting on an n-dimensional cohomology manifold (n-cm) M over the integers in such a way that all the non-fixed orbits are of the same dimension k, then either G is of rank one and all isotropy subgroups are finite, or the dimension of the fixed set F(G, M) is n - k - 1. In the latter case we also know from Chapter XV of [1] that F(G, M) is an (n - k - 1)-cm, the orbit space M/G is an (n - k)-cm with boundary F(G, M), there exist local cross-sections for the orbits of G near F(G, M), and the non-fixed orbits are k-spheres (see [2]).

The importance of this result lies in the fact that the set of those points x in B, the singular set, for which the hypotheses of the theorem are true for the action of G_x on a slice at x, is a dense open subset of B. The proof of the theorem is quite different from that of Borel's and Conner's result, the difficulty lying in the fact that, in order to obtain a result which is hereditary with respect to taking slices, we cannot make any hypotheses of simple connectivity, whereas the proofs of Borel and Conner make strong use of such a hypothesis.

We use the notation of [1], throughout. B denotes the set of points on singular orbits, that is, orbits of less than the highest dimension. E denotes the set of points on exceptional orbits of highest dimension, so that $M - (B \cup E)$ is the set of points on principal orbits. Unless otherwise specified, H denotes a principal isotropy group, T_0 is a maximal torus of G, and $T \subset T_0$ a maximal torus of H. N(K) and Z(K) denote, respectively, the normalizer and the centralizer of K in G, and K^0 denotes the identity component of K.

We assume that the reader is familiar with the theory of transformation groups, or more specifically with the material in Chapters I, V, VIII, IX, and XIII of [1]. It is also desirable, but not essential, that the reader be familiar with the author's Chapter XV of [1], particularly because it is the main result of that chapter that gives the present work its force. One of the main tools used in the present paper is the remarkable formula of Borel [1, XIII] relating the dimension of the fixed point set of a torus acting on an n-cm to the dimensions of the fixed point sets of the subtori of codimension one.

2. THE MAIN THEOREM

Our main result is the following:

THEOREM. *Let* G *be a compact Lie group acting effectively on an* n-cm M *over* Z *such that every point of* F(G, M) *has a countable system of neighborhoods in* M. *Assume that* $F(G, M) \neq \emptyset$, *and that for* $x \in M - F(G, M)$ *we have* $\dim G(x) = k$,

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Received September 3, 1960.
This research was sponsored by NSF grant No. G-10700.