THE EMBEDDING OF CERTAIN METRIC FIELDS

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1. INTRODUCTION

Mazur's Theorem, first stated in [5], can be formulated for the commutative case in the following way: *If* K *is a metric field which contains the real numbers and is such that the norm of* x *equals the ordinary absolute value of* x *whenever* x *is a real number, then* K *is the real field or the complex field.* In Section 4 of this paper, some results are obtained in which K is no longer assumed to contain the real field, and the condition on the norm is assumed only for a portion of the prime field of K; when K does not contain the real field, it might be one of the subfields of the complex field other than the reals or complexes; therefore our results show only that K is some subfield of the complex field.

For instance, Corollary 2 of Theorem 5 asserts that if K is a metric field such that the norm of $n \cdot e$ equals n, for every natural number n which is sufficiently large, then K is isomorphic to a subfield of the complex field. (Here e denotes the unit element of K, and $n \cdot e$ is the n-fold sum $e + \cdots + e$.) Similarly, Theorem 7 asserts that if K is a metric field of characteristic zero, such that the norm of x equals the ordinary absolute value of x whenever x is a positive rational number in K that is sufficiently small, then K is a subfield of the complex field.

In order to obtain these results, we develop in Section 3 some generalizations of Theorem 1 of [2]. These generalizations allow us to obtain, from a pseudonorm N which is sufficiently well-behaved on a semigroup A, a pseudonorm N' closely related to N, having the same desirable properties as N, and such that

$$N'(cx) = N'(c)\, N'(x)$$

for all c in A and for all x (in this paper, a *semigroup* is understood to be a nonempty set, contained in a ring, which is closed under the ring multiplication).

2. PSEUDONORMS AND SUBORDINATE PSEUDONORMS

The terminology and notation employed in [2] are assumed known. See also [1] and [4] for further remarks about norms and metric rings.

A pseudonorm N is said to be *stable* at an element c, and c is said to be N-*stable*, if $N(\cdots cx \cdots) = N(\cdots xc \cdots)$ for all x; if N is stable at every element of a set A, then A is said to be N-*stable*, and we say that N is *stable on* A. Pseudo absolute values are stable on the entire ring, and every pseudonorm is stable on the center.

If N is a pseudonorm of a ring R and c is an element of R such that $N(c^r) = N(c)^r$ for $r = 1, 2, \cdots$, then we say that N is *power multiplicative at* c and that c is N-*power multiplicative*. A pseudo absolute value is power multiplicative at all elements, for instance.

Under certain circumstances it is possible to replace a pseudonorm by a subordinate pseudonorm having special properties. First, we note that if N is a

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