TRANSFORMATION GROUPS ON A $K(\pi, 1)$, II

P. E. Conner

1. INTRODUCTION

This note is concerned with the study, by methods introduced in [3], of involutions on a finite-dimensional $K(\pi, 1)$. For the sake of simplicity, we shall restrict our discussion to involutions, although our methods apply equally well to cyclic transformations of prime order.

Throughout this note, we denote by (T, X) an involution with at least one fixed point. The fixed point set of (T, X) is denoted by $F \subset X$. We assume that X is locally compact, connected, separable metric, and locally contractible. We denote by Y the universal covering space of X, and

$$\alpha: Y \to X$$

is the covering map. If $\pi = \pi_1(X)$ is the fundamental group of X, then (π, Y) is the action of π on Y defining a principal fibre structure in the covering space. The elements of π are denoted by the letter σ, with suitable subscripts when it is necessary to consider more than one element at a time.

An *involution with base point*, $(T, (X, x))$, is an involution for which a fixed point x is chosen as the reference point. An involution with base point induces an automorphism $T_*: \pi_1(X, x) \to \pi_1(X, x)$ of period 2. It was shown in [3] that an involution with a base point $(T, (X, x))$ admits a covering involution (t, Y) such that $\alpha(t(y)) = T(\alpha(y))$ and

$$t(\sigma(y)) = T_*(\sigma)(t(y)),$$

for any element $\sigma \in \pi$ and any point $y \in Y$.

2. PRELIMINARY FORMULAS

We let $(T, (X, x))$ be a fixed involution with base point, and we let (t, Y) be the covering involution. We define a subset $C \subset \pi = \pi_1(X, x)$ by

$$C = \{\, \sigma \mid \sigma(y) = t(y) \text{ for some } y \in Y \} .$$

Furthermore, for $\sigma \in C$, let

$$F(\sigma) = \{\, y \mid y \in Y,\ \sigma(y) = t(y) \} .$$

The importance of the sets $F(\sigma)$ lies in the fact that $\alpha(y) \in F$ if and only if $y \in F(\sigma)$ for some $\sigma \in C$.

(2.1) LEMMA. *If* $\sigma \in C$, *then* $T_*(\sigma) = \sigma^{-1}$, *and the map* $t_\sigma: Y \to Y$ *defined by* $y \to \sigma^{-1}(t(y))$ *is an involution.*

Received July 2, 1959.