DIVERGENCE OF RANDOM POWER SERIES

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1. INTRODUCTION

Let $\phi_n(t)$ $(n = 0, 1, 2, \cdots)$ be the Rademacher functions, that is, let

$$\phi_n(t) = (-1)^j \text{ for } j/2^n \leq t < (j+1)/2^n \qquad (j = 0, 1, \cdots, 2^n - 1;\ n = 0, 1, 2, \cdots).$$

Given any sequence of complex numbers $\{a_n\} = \{a_0, a_1, a_2, \cdots\}$, we denote by $\mathscr{F}\{a_n\}$ the family of power series

$$P(z;\, t) = \sum_{n=0}^{\infty} \phi_n(t)\, a_n\, z^n \qquad (0 \leq t < 1). \tag{1}$$

It is well known that if

$$\sum_{n=0}^{\infty} |a_n|^2 = \infty, \tag{2}$$

then *almost all* series (1) diverge *almost everywhere* on $|z| = 1$. Here *almost all* refers to the set of t (in the usual Lebesgue sense), while *almost everywhere* refers to the set of z on the circumference of the unit circle (again in the usual sense).

Only recently [1] was it observed that some nontrivial interesting assertions similar to the above with *almost everywhere* replaced by *everywhere* can be made. Here a new result of this kind, going beyond that indicated in [1], will be established.

2. STATEMENT OF RESULTS

Our main result is the following

THEOREM. *Let $\{c_n\}_{n=0}^{\infty}$ be a monotone sequence of positive numbers tending to zero and satisfying the condition*

$$\limsup_{n \to \infty} \frac{\sum_{j=0}^{n} c_j^2}{\log 1/c_n} > 0. \tag{3}$$

If $\{a_n\}_0^{\infty}$ is a sequence of complex numbers satisfying the condition

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