A GENERALIZED MANIFOLD

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INTRODUCTION

While generalized manifolds share many of the properties of classical manifolds, there has been a general question: How far can a generalized manifold get away from classical manifolds? In particular, it does not seem to have been settled whether or not there exists a generalized manifold that is not locally euclidean at any point. A generalized manifold that fails to be locally euclidean at a single point can easily be obtained by shrinking a bad arc in S^3 to a point [3]. This being the case, one readily suspects that a generalized manifold that is not locally euclidean at any point can be obtained by putting bad arcs densely in S^3 and shrinking each to a point. In the present note, we show that this is indeed the case. In the course of the construction, however, care must be taken so that the decomposition space is still a Hausdorff space and is also finite-dimensional. The first requirement is fulfilled if we put in arcs such that the arcs together with the points not on any of the arcs form an upper-semicontinuous decomposition (see [8] for definition) of S^3.

1. THE CONSTRUCTION OF AN UPPER-SEMICONTINUOUS DECOMPOSITION G

We denote by E^3 and S^3 the euclidean 3-space and 3-sphere, respectively. By a 3-cell we mean a homeomorph of the unit sphere together with its interior in E^3. By the boundary and the interior of a 3-cell we mean the parts that correspond to the unit sphere and its interior, and they will be denoted, sometimes, by Bd and Int, respectively.

It is known [4] that there exists an arc A in E^3 such that E^3 - A is not simply connected. It is easy to see that such an A can be put into any pre-assigned open subset of E^3.

Let $\mathfrak{U}^0 = \{U_1^0, U_2^0, \cdots, U_{k_0}^0\}$ be an open covering of S^3 such that each U_i^0 is the interior of a 3-cell $\overline{U}_i^0$ of diameter less than 1. Let F^0 be the union of Bd U_i^0. In each U_i^0, we can find an arc A_i^0 such that (1) the arcs A_i^0 are pairwise disjoint, (2) each A_i^0 is situated in U_i^0 in the same manner as A is in E^3, and (3) no A_i^0 meets F^0. There exists a positive number $d_1 < 1/2$ such that no $3d_1$-neighborhood of any A_i^0 meets F^0 or any other A_j^0.

Let $\mathfrak{U}^1 = \{U_1^1, U_2^1, \cdots, U_{k_1}^1\}$ be an open overing of S^3 such that each U_i^1 is the interior of a 3-cell $\overline{U}_i^1$ of diameter less than d_1. Let F^1 denote the union of the Bd U_i^1. In each U_i^1, we can find an arc A_i^1 such that (1) the arcs A_p^0 and A_q^1 are pairwise disjoint, (2) each A_i^1 is situated in U_i^1 in the same way as A is in E^3, and (3) no A_i^1 meets F^0 or F^1. There exists a positive number $d_2 < \min(d_1, 1/4)$ such that no $3d_2$-neighborhood of any A_p^0 or A_q^1 meets any other $A_{p'}^0$ or $A_{q'}^1$ and no $3d_2$-neighborhood of A_q^1 meets F^0 or F^1.

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