THE CONSTRUCTION OF HADAMARD MATRICES

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An Hadamard matrix H is a (1, -1)-matrix of order 4n such that

$$HH^T = 4nI_{4n}, \tag{1}$$

where I_{4n} is the identity matrix of order 4n. The result of this paper is as follows.

THEOREM. *An Hadamard matrix of order* 4n *can be constructed if there exists a transitive permutation group of degree* 4n - 1 *and odd order whose subgroups leaving one element fixed have three transitivity sets each.*

Suppose we can find a (0, 1)-matrix A of order 4n - 1 satisfying

$$AA^T = nI + (n-1)J, \tag{2}$$

where I is the identity matrix, and J is the matrix with 1 in every position, of order 4n - 1. Then the bordered matrix

$$H = \begin{pmatrix} 1 & 1 \quad \cdots \quad 1 \\ 1 & \\ \cdot & \\ \cdot & 2A - J \\ \cdot & \\ 1 & \end{pmatrix}$$

satisfies (1). We shall prove that, given the permutation group of the theorem, we can construct a matrix A satisfying (2).

Let G be the permutation group of the theorem, and suppose it permutes the integers 1, 2, ⋯, 4n - 1. For each g in G, let P(g) be the permutation matrix of order 4n - 1 with 1 in the (i, j)-position if i = g(j).

Let $\mathscr{A}$ be the algebra of matrices of order 4n - 1 which commute with every P(g). If (x_{ij}) is any such matrix, then

$$(x_{ij}) = P(g)^{-1}(x_{ij})P(g) = (x_{g(i)g(j)}) \qquad (\text{all } g \in G),$$

and therefore (x_{ij}) is characterized by

$$x_{ij} = x_{g(i)g(j)} \qquad (\text{all } i, j = 1, 2, \cdots, 4n-1 \text{ and all } g \in G). \tag{3}$$

The equivalence

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