GROUPS ON R^n OR S^n

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1. INTRODUCTION

Throughout this paper G will be a compact connected Lie group acting on a manifold M which is either R^n or S^n, that is, euclidean n-space or the n-sphere. Furthermore the group is assumed to act differentiably, by which is meant that each homeomorphism of M is of class C^1 in the ordinary differentiable structure of M. The space M is divided into certain disjoint subsets as follows. If r is the highest dimension of any orbit, let B be the set of points on orbits of dimension less than r. The set B is closed, and it is known [1] that $\dim B \leq n - 2$. Let D be the set of points x satisfying

a) $\dim G(x) = r$,

b) in every neighborhood of x there is a point y such that G_y, the isotropy group at y, has fewer components than G_x.

Any orbit in D is called an exceptional orbit of highest dimension. Near such an orbit $G(x)$, there is another highest-dimensional orbit $G(y)$ which "wraps around" $G(x)$ more than once.

Let U be the set of all points on orbits of highest dimension which are not in D. Then $x \in U$ if and only if

a) $\dim G(x) = r$,

b) for all y in some neighborhood of x, G_x and G_y have the same number of components.

The sets B, D, U are invariant and disjoint, and

$$M = B \cup D \cup U;$$

B is closed, $B \cup D$ is closed, and U is open. For the case at hand [2],

$$\dim D \leq n - 2.$$

The orbits of M can be made into a space M^*, called the orbit space; and M^* contains the disjoint sets B^*, D^*, U^* which are the images of B, D, U under the map from M to M^*. The map from M to M^* is denoted by T.

This paper studies some of the properties of these sets, and it proves the following theorems.

THEOREM 1. *Let a compact connected Lie group* G *act differentiably on* $M = R^n$ *or* S^n. *Then* $U^* \cup D^*$ *is simply connected.*

COROLLARY. *Under the hypothesis of Theorem* 1, *let* $\dim D \leq n - 3$. *Then* U^* *is simply connected.*

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