ASYMPTOTIC SOLUTIONS WITH RESPECT TO A PARAMETER OF ORDINARY DIFFERENTIAL EQUATIONS HAVING A REGULAR SINGULAR POINT

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1. INTRODUCTION

We examine the asymptotic behavior for large $|\lambda|$ of solutions of the differential equation

$$\frac{d^2u}{dz^2} + [\lambda^2 q_0(z) + \lambda q_1(z) + F(z, \lambda)]u = 0, \tag{1.1}$$

with z restricted to a closed simply connected region $\mathfrak{D}$ containing the origin in its interior, and under the principal assumption that *the functions* zq_i *and* $z^2F(z, \lambda)$ *are analytic in* $\mathfrak{D}$. We also assume that $F(z, \lambda)$ *is analytic in* λ *when* $|\lambda| > N$. Thus

and near the origin,

$$(1.2)\quad \begin{cases} \text{a)} & F(z, \lambda) = \sum_0^\infty f_i(z)\lambda^{-i} \qquad (|\lambda| > N); \\ \text{b)} & zq_i(z) = \sum_0^\infty q_{ik}z^k, \qquad (i = 0, 1), \\ \text{c)} & z^2 f_i(z) = \sum_0^\infty f_{ik}z^k. \end{cases}$$

We lose no generality by assuming that $q_{00} = 1$.

The main conclusions of the paper are Theorems 1, 2, and 3 in Sections 6 and 7. We observe, from these conclusions, that our theory is a special case of a general theory which also is applicable to equations of the type

$$\frac{d^2u}{dz^2} + \lambda^2 Q(z, \lambda)u = 0, \tag{1.3}$$

where $Q(z, \lambda) = \Sigma_0^\infty q_i(z)\lambda^{-i}$ and $q_0(z) = z^k\Sigma_0^\infty c_n z^n$ with $k = 0$ or $k = 1$. These equations were considered by Langer in [2]. Unfortunately, the general theory does not extend to values of k other than -1, 0, and 1.

The present work is motivated partly by the fact that the differential equations for several of the special functions, for example, the Whittaker, Legendre, and

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