A SCALAR TRANSPORT EQUATION, II

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1. INTRODUCTION

This is the second paper in a series treating the formulation of a nonlinear integro-differential equation which occurs in a variety of physical problems, and discussing the existence and properties of its solutions. The first paper [2] was concerned with the equation

$$(1)\qquad \begin{aligned} \frac{\partial f(x, t)}{\partial t} &= \frac{1}{2}\int_0^x f(y, t) f(x-y, t)\phi(y, x-y)\,dy - f(x, t)\int_0^\infty f(y, t)\phi(x, y)\,dy \\ &+ \int_x^\infty f(y, t)\psi(y, x)\,dy - \frac{f(x, t)}{x}\int_0^x y\,\psi(x, y)\,dy \qquad (x, t \geq 0), \end{aligned}$$

where $f(x, 0)$, $\phi(x, y)$ and $\psi(x, y)$ are given and $f(x, t)$ is to be determined. It was shown that under certain hypotheses equation (1) possesses a unique solution $f(x, t)$.

The present paper will treat a more general equation in which $\phi(x, y, t)$ and $\psi(x, y, t)$ replace $\phi(x, y)$ and $\psi(x, y)$, respectively. The following is the main result obtained.

THEOREM 1. *Let* $f(x, y)$, $\phi(x, y, t)$ *and* $\psi(x, y, t)$ *be functions which satisfy the following hypotheses:*

(H_1) $f(x, 0)$ *is nonnegative, bounded, continuous and integrable, and*

$$\int_0^\infty x f(x, 0)\,dx < \infty;$$

(H_2) $\phi(x, y, t)$ *is nonnegative and bounded,*

$\phi(x, y, t) = \phi(y, x, t)$,

$\phi(x, y, t)$ *is continuous with respect to* x, y, *and* t, *and continuity in* t *is uniform with respect to* x *and* y;

(H_3) $\psi(x, y, t)$ *is nonnegative and bounded,*

$$\int_0^x \psi(x, y, t)\,dy < E - 1 < \infty,$$

$\psi(x, y, t)$ *is continuous with respect to* x, y *and* t, *and continuity in* t *is uniform with respect to* x *and* y; *also, the function* $\frac{1}{x}\int_0^x y\,\psi(x, y, t)\,dy$ *is bounded*

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