COHOMOLOGY THEORY IN TOPOLOGICAL GROUPS

by

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Introduction. For a topological group Q, there are two obviously different cohomology theories which have been established in the mathematical literature, namely, the cohomology theory of Q as a topological space and that of Q as an abstract group [6] [2]. The initial purpose of the present work is to study the possible relations between these theories.

In Chapter I, for a topological group Q which operates on a topological abelian group G, three kinds of cohomology groups of Q over the coefficient group G are introduced, namely, the cohomology groups, the cohomology groups with empty supports, and the reduced cohomology groups. The methods used here are more or less analogous to those of Eilenberg and MacLane [6]. The first kind of the cohomology groups of Q over G reduce to the cohomology groups of the abstract group Q over G if Q is discrete, while the reduced cohomology groups of Q over G are closely related to those of Q as a topological space. In fact, it is proved in Chapter II that, if Q is compact and connected and G is a finite dimensional vector group on which Q operates simply, then the reduced cohomology groups of Q over G are isomorphic with the Cech cohomology groups of Q as a topological space over the abstract group G.

In Chapter III, we define a local cohomology theory of a local group Q over a local abelian group G on which Q operates. It is proved that, if a topo-