CONFORMAL MAPPING OF A JORDAN REGION WHOSE BOUNDARY HAS POSITIVE TWO-DIMENSIONAL MEASURE

by

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Jordan curves which pass through plane sets of positive two-dimensional measure have been constructed by Osgood [4] and Kline [1]. The present note is concerned with the relation between such Jordan curves and the following general problem. Let R be the finite region in the w-plane bounded by a Jordan curve C, and let E_w be a set of points on C. Let the function f(z) be continuous and univalent in $|z| \leq 1$ and holomorphic in $|z| < 1$, and let it map the set $|z| \leq 1$ upon the closure of R. The general problem (still unsolved) is that of finding necessary and sufficient conditions on E_w in order that the image E_z on the circle $|z| = 1$ have linear measure zero. A sufficient condition is that no points of E_w be accessible from R by rectifiable arcs ([2], [5], [6]). By means of this sufficient condition, the following result will be established.

THEOREM. There exists a Jordan region R such that, under a conformal mapping of R onto the region $|z| < 1$, a certain set lying on the boundary of of R and having positive two-dimensional measure is mapped into a set of linear measure zero on $|z| = 1$.

Let A be an isosceles right triangle in the w-plane. Let B_0 denote the intersection of A with a closed strip bounded by two parallel lines, one of which passes through the hypotenuse of A, while the