THE LOTOTSKY METHOD FOR EVALUATION OF SERIES

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1. INTRODUCTION

A. V. Lototsky (or Lotockii) [5] has recently introduced a method for evaluation of divergent series which seems to be new and to have fundamental significance which may make it rival in importance the classic methods of Cesàro, Abel, Euler and Knopp, Borel, and others. The method involves a triangular matrix transformation of the standard form

$$\sigma_n = \sum_{k=1}^{n} a_{nk} s_k \tag{1.1}$$

by which a given series $u_1 + u_2 + u_3 + \cdots$ with partial sums $s_1 = u_1$, $s_2 = u_1 + u_2$, $\cdots$ is evaluable to σ if $\sigma_n \to \sigma$ as $n \to \infty$.

For each $n = 1, 2, 3, \cdots$, let $p_n(x)$ be the polynomial of degree n defined by

$$p_n(x) = x(x+1)(x+2)\cdots(x+n-1), \tag{1.2}$$

and let the constants $p_{n1}, p_{n2}, \cdots, p_{nn}$ be defined by

$$p_n(x) = p_{n1}x + p_{n2}x^2 + p_{n3}x^3 + \cdots + p_{nn}x^n. \tag{1.3}$$

To simplify our work in some places, we let $p_{nk} = 0$ when $k < 1$ and when $k > n$. Letting $a_{nk} = p_{nk}/n!$ and

$$\sigma_n = \sum_{k=1}^{n} \frac{p_{nk}}{n!} s_k, \tag{1.4}$$

we shall call a series $u_1 + u_2 + \cdots$ and its sequence $s_1, s_2, \cdots$ of partial sums *evaluable* L to σ if $\sigma_n \to \sigma$ as $n \to \infty$.

Numerous properties of this Lototsky method are obtained. Because the paper [5] of Lototsky appears in a periodical that is not always readily accessible, no acquaintance with it is assumed, and the connections between the present paper and [5] are explained in such a way that they can be understood without reference to [5].

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