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Compactness of Composition Operators on the Bloch Space in Classical Bounded Symmetric Domains

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1. Introduction

Let $\mathcal{D}$ be a bounded homogeneous domain in $\mathbb{C}^N$. The class of all holomorphic functions with domain $\mathcal{D}$ will be denoted by $H(\mathcal{D})$. Let ϕ be a holomorphic self-map of $\mathcal{D}$. For $f \in H(\mathcal{D})$, we denote the composition $f \circ \phi$ by $C_\phi f$ and call C_ϕ the composition operator induced by ϕ.

Let $K(z, z)$ be the Bergman kernel function of $\mathcal{D}$. The Bergman metric $H_z(u, u)$ in $\mathcal{D}$ is defined by

$$H_z(u, u) = \frac{1}{2} \sum_{l,k=1}^{N} \frac{\partial^2 \log K(z, z)}{\partial z_l \partial \bar{z}_k} u_l \bar{u}_k,$$

where $z \in \mathcal{D}$ and $u = (u_1, \dots, u_N) \in \mathbb{C}^N$.

Following Timoney [T], we say that $f \in H(\mathcal{D})$ is in the Bloch space $\beta(\mathcal{D})$ if

$$\|f\|_{\beta(\mathcal{D})} = \sup_{z \in \mathcal{D}} Q_f(z) < \infty, \tag{1}$$

where

$$Q_f(z) = \sup\left\{ \frac{|\nabla f(z) u|}{H_z^{1/2}(u, u)} : u \in \mathbb{C}^N - \{0\} \right\}$$

and where $\nabla f(z) = \left(\frac{\partial f(z)}{\partial z_1}, \dots, \frac{\partial f(z)}{\partial z_N}\right)$ and $\nabla f(z) u = \sum_{l=1}^{N} \frac{\partial f(z)}{\partial z_l} u_l$.

Let D be the unit disk in $\mathbb{C}$. Madigan and Matheson [MM] proved that C_ϕ is always bounded on $\beta(D)$. They also gave the sufficient and necessary conditions for C_ϕ to be compact on $\beta(D)$.

More recently, Shi and Luo [SL] proved that C_ϕ is always bounded on $\beta(\mathcal{D})$ and gave a sufficient condition for C_ϕ to be compact on $\beta(\mathcal{D})$, where $\mathcal{D}$ is a bounded homogeneous domain in $\mathbb{C}^N$.

By using Cartan's list, all irreducible bounded symmetric domains are divided into six types. The first four types of irreducible domains are called the classical bounded symmetric domains. The other two types, called exceptional domains, consist of one domain each (a 16- and a 27-dimensional domain).

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