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CR Maps and Point Lie Transformations

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1. Introduction

This paper concerns the following well-known result (see Chern and Moser [3]) of geometric complex analysis: Any biholomorphic map between two real analytic Levi nondegenerate hypersurfaces in $\mathbb{C}^{n+1}$ $(n > 0)$ is uniquely determined by its 2-jet at fixed point. Moser's proof is based on his general theory of normal forms for Levi nondegenerate hypersurfaces. We present a new geometric approach to the problem that allows us to deduce Moser's result from a general assertion concerning point Lie transformations of certain second-order PDE systems. The Segre family of a Levi nondegenerate hypersurface is a general solution of such a system, and every biholomorphism of such a hypersurface is a Lie symmetry of this system. Our approach is mostly inspired by the ideas of Webster [10] as well as the works of Diederich and Webster [5] and Diederich and Fornæss [4].

Our main result is the following.

Theorem 1.1. *Any holomorphic point transformation between two holomorphic completely integrable systems $D^{(2)}u = F(x, u, D^{(1)}u)$ and $D^{(2)}u = \hat{F}(x, u, D^{(1)}u)$ with one dependent variable and n independent variables is determined by its 2-jet at a fixed point. The set of all such transformations can be parameterized by at most $n^2 + 4n + 3$ complex parameters.*

The terminolgy will be explained in the next section. The infinitesimal version of this theorem has been established by the author in [9].

We stress that PDE systems defining Segre families of real analytic hypersurfaces form a highly special subclass of PDE systems considered in our result. From this point of view, the study of point transformations of PDE systems is a substantially more general problem. We hope that our approach will be useful for both the CR geometry and the geometry of differential equations.

2. Preliminaries

In this section we establish a correspondence between the geometry of real analytic CR structures and completely integrable PDE systems.

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