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Quotients of Divisorial Toric Varieties

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Introduction

A frequently occurring question in algebraic geometry is whether an algebraic group action $G \times X \to X$ admits a categorical quotient—that is, a regular map $X \to Y$ that is universal with respect to G-invariant regular maps $X \to Z$. For example, moduli functors are often co-represented by categorical quotients. In general, it is a difficult problem to decide whether a categorical quotient exists. Some counterexamples for actions of the multiplicative group $\mathbb{C}^*$ are presented in [4].

As these examples show, difficulties already arise with subtorus actions on toric varieties. Such actions have been investigated by several authors, mainly focusing on the much more restrictive concept of a good quotient (see e.g. [13; 16; 21]). The description of toric varieties in terms of rational fans relates the problem of constructing quotients to problems of combinatorial convexity. Hence the class of toric varieties serves as a testing ground for more general ideas.

Let X be a toric variety and let H be a subtorus of the big torus of X. Our approach to categorical quotients for the induced action of H on X is to consider the problem in suitable subcategories. A first step is to construct a quotient in the category of toric varieties itself: in [2], we showed that there always exists a *toric quotient*

$$p\colon X \to X/_{\mathrm{tq}}H.$$

This is a toric morphism that is universal with respect to H-invariant toric morphisms. The essential part of the proof is an explicit algorithm in terms of combinatorial data. The toric quotient is a canonical starting point for quotients in further categories. For example, in [3] we gave an explicit method to decide by means of the toric quotient when a subtorus action on a quasiprojective toric variety admits a categorical quotient in the category of quasiprojective varieties.

In this paper we give a considerable generalization of the results of [3]; namely, we solve the analogous problem in the category of divisorial varieties. Recall that an irreducible variety X is called *divisorial* if every point $x \in X$ has an affine neighborhood of the form $X \setminus \mathrm{Supp}(D)$ with an effective Cartier divisor D on X (see e.g. [10] and [8, II.2.2]).

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