Michigan Math. J. 50 (2002)

Convexity Properties for Cycle Spaces

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1. Introduction

In this article we study compact q-cycles on a complex reduced analytic space X, mainly in the case where q is the maximal dimension of a compact (irreducible) analytic subset of X.

We first give a result that generalizes a classical result due to Norguet and Siu [18] about finiteness of compact hypersurfaces in a p-convex manifold; it gives a suitable sufficient condition for X to have only finitely many irreducible compact q-cycles.

Theorem 1. *Let X and Y be complex spaces such that X is contained in Y as a locally closed analytic subset. Suppose that:*

(a) $H^q(X, \Omega_X^q)$ *has finite dimension over $\mathbb{C}$, say N; and*
(b) $H^{q+1}(Y, \mathcal{F}) = 0$ *for every coherent subsheaf $\mathcal{F} \subset \Omega_Y^q$.*

Then X has at most N compact irreducible analytic subsets of dimension q.

We then study the convexity properties of the space of compact q-cycles $\mathcal{C}_q(X)$.

Theorem 2. *Let X be a cohomologically q-complete complex space that is Kählerian and $(q + r)$-convex for some nonnegative integer r. Then $\mathcal{C}_q(X)$ is r-complete with corners.*

This looks like a nice "convexity transfer", but it is quite weak because the r-convexity with corners is not so restrictive for $r > 0$. The method is similar to the one used in [18] but requires us to work with r-plurisubharmonic functions (see Section 3.1 for definitions) and to prove an approximation result by functions that are r-convex with corners.

Theorem 3. *Let Z be a complex space admitting a continuous exhaustion function φ that is q-plurisubharmonic. If Z belongs to $\mathcal{S}_0$, then Z is q-complete with corners.*

Note. $\mathcal{S}_0$ is the class of complex spaces such that, on every relatively compact open subset, there exist continuous strongly plurisubharmonic functions. For instance,

Received January 4, 2001. Revision received August 6, 2001.