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On the L^p Boundedness of Marcinkiewicz Integrals

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1. Introduction and Results

Let $n \geq 2$ and let S^{n-1} be the unit sphere in $\mathbb{R}^n$ equipped with the normalized Lebesgue measure $d\sigma$. Let $b(\cdot) \in L^\infty(\mathbb{R}_+)$ and let Ω be a homogeneous function of degree zero on $\mathbb{R}^n$ (which is then naturally identified with a function on S^{n-1}) satisfying $\Omega \in L^1(S^{n-1})$ and

$$\int_{S^{n-1}} \Omega(y)\, d\sigma(y) = 0. \tag{1.1}$$

For a suitable mapping $\Phi\colon \mathbb{R}^n \to \mathbb{R}^d$, we define the Marcinkiewicz integral operator $\mu_{\Phi,\Omega,b}$ on $\mathbb{R}^d$ by

$$\mu_{\Phi,\Omega,b}(f)(x) = \left(\int_0^\infty |F_{\Phi,t}(x)|^2 \frac{dt}{t^3} \right)^{1/2}, \tag{1.2}$$

where

$$F_{\Phi,t}(x) = \int_{|y|\leq t} \frac{\Omega(y)}{|y|^{n-1}} b(|y|) f(x - \Phi(y))\, dy. \tag{1.3}$$

If $n = d$, $\Phi(y) = (y_1, y_2, \dots, y_n)$, and $b \equiv 1$, then we shall simply denote the operator $\mu_{\Phi,\Omega,b}$ by μ_Ω.

The main purpose of this paper is to study the L^p boundedness of the operators $\mu_{\Phi,\Omega,b}$. The operator μ_Ω was introduced by Stein [S1]. He proved that if Ω satisfies a Lip_α $(0 < \alpha \leq 1)$ condition on S^{n-1}, then μ_Ω is of type (p, p) for $1 < p \leq 2$ and of weak type $(1, 1)$. Subsequently Benedek, Calderón, and Panzone [BCP] showed that if Ω is continuously differentiable on S^{n-1} then μ_Ω is of type (p, p) for $1 < p < \infty$. In a more recent paper [DFP] we obtained the L^p boundedness of μ_Ω under the substantially weaker assumption that $\Omega \in H^1(S^{n-1})$. In fact, it was proved in [DFP] that the operator $\mu_{\mathbf{1},\Omega,b}$ is bounded on $L^p(\mathbb{R}^n)$ provided that $\Omega \in H^1(S^{n-1})$ and $b(\cdot) \in L^\infty(\mathbb{R}_+)$. Here $\mathbf{1}$ represents the identity mapping from $\mathbb{R}^n$ to itself and $H^1(S^{n-1})$ denotes the Hardy space on the unit sphere that contains $L \log^+ L(S^{n-1})$ as a proper subspace (see Section 3 for its definition).

In this paper we shall establish the L^p boundedness of $\mu_{\Phi,\Omega,b}$ for several classes of mapping Φ with rough kernels Ω, mirroring recent developments in the theory

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