Michigan Math. J. 50 (2002)

On Ideals in H^∞ Whose Closures Are Intersections of Maximal Ideals

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Dedicated to Professor Kôzô Yabuta on his sixtieth birthday

1. Introduction

Let H^∞ be the Banach algebra of bounded analytic functions on the open unit disk D. We denote by $M(H^\infty)$ the maximal ideal space of H^∞, the set of nonzero multiplicative linear functionals of H^∞ endowed with the weak*-topology of the dual space of H^∞. Identifying a point in D with its point evaluation, we think of D as a subset of $M(H^\infty)$. For $\varphi \in M(H^\infty)$, put $\operatorname{Ker}\varphi = \{f \in H^\infty; \varphi(f) = 0\}$. Then $\operatorname{Ker}\varphi$ is a maximal ideal in H^∞, and for a maximal ideal I in H^∞ there exists $\psi \in M(H^\infty)$ such that $I = \operatorname{Ker}\psi$. For $f \in H^\infty$, the function $\hat{f}(\varphi) = \varphi(f)$ on $M(H^\infty)$ is called the *Gelfand transform* of f. We can identify f with $\hat{f}$, so that we think of H^∞ as the closed subalgebra of continuous functions on $M(H^\infty)$. Let L^∞ be the Banach algebra of bounded measurable functions on ∂D. The maximal ideal space of L^∞ will be denoted by $M(L^\infty)$. We may think of $M(L^\infty)$ as a subset of $M(H^\infty)$. Then $M(L^\infty)$ is the Shilov boundary of H^∞, that is, the smallest closed subset of $M(H^\infty)$ on which every function in H^∞ attains its maximal modulus. For a subset E of $M(H^\infty)$, we denote the closure of E by $\bar{E}$. A nice reference for this subject is [4].

For $f \in H^\infty$, there exists a radial limit $f(e^{i\theta})$ for almost everywhere. Let h be a bounded measurable function on ∂D such that $\int_0^{2\pi} \log|h|\, d\theta/2\pi > -\infty$. Put

$$f(z) = \exp\left(\int_0^{2\pi} \frac{e^{i\theta}+z}{e^{i\theta}-z} \log|h(e^{i\theta})| \frac{d\theta}{2\pi}\right), \quad z \in D.$$

A function of this form is called *outer*, and $|f(e^{i\theta})| = |h(e^{i\theta})|$ almost everywhere. A function $u \in H^\infty$ is called *inner* if $|u(e^{i\theta})| = 1$ a.e. on ∂D. For a sequence $\{z_n\}_n$ in D with $\sum_{n=1}^\infty (1-|z_n|) < \infty$, there corresponds a Blaschke product

$$b(z) = \prod_{n=1}^\infty \frac{-\bar{z}_n}{|z_n|} \frac{z-z_n}{1-\bar{z}_n z}, \quad z \in D.$$

A Blaschke product is called *interpolating* if, for every bounded sequence of complex numbers $\{a_n\}_n$, there exists $h \in H^\infty$ such that $h(z_n) = a_n$ for every n. For a nonnegative bounded singular measure μ ($\mu \neq 0$) on ∂D, let

Received July 18, 2000. Revision received August 29, 2001.

Supported by Grant-in-Aid for Scientific Research (no. 10440039), Ministry of Education, Science, Sports and Culture.