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An Analog of the Classical Invariant Theory for Lie Superalgebras, II

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This paper is a detailed exposition of [S3] with several new results added. It also complements and refines the results of [S2]. Meanwhile there has appeared a paper [J1] where a particular case is considered but in more detail and where other references are offered; see also [J2] and [Y].

1. Preliminaries

In what follows, $\mathfrak{S}_k$ stands for the symmetric group on k elements. Let λ be a partition of the number k and let t be a λ-tableau. Recall that t is called *standard* if the numbers in its rows and columns grow from left to right and downward. Denote by C_t the column stabilizer of t, and let R_t be its row stabilizer. We further set

$$e_t = \sum_{\tau \in C_t;\, \sigma \in R_t} \varepsilon(\tau)\sigma\tau, \qquad \tilde{e}_t = \sum_{\tau \in C_t;\, \sigma \in R_t} \varepsilon(\tau)\tau\sigma. \tag{0.1}$$

Let $\mathbb{N}$ be the set of positive integers, let $\bar{\mathbb{N}}$ be another, "odd", copy of $\mathbb{N}$, and let $\mathbb{M} = \mathbb{N} \coprod \bar{\mathbb{N}}$ be ordered so that each element of the "even" copy ($\mathbb{N}$) is smaller than any element from the "odd" copy; inside of each copy, the order is the natural one. We will call the elements from $\mathbb{N}$ "even" and those from $\bar{\mathbb{N}}$ "odd", so we can encounter an "even" odd element and so forth.

Let I be the sequence of elements from $\mathbb{M}$ of length k. We fill in the tableau t with elements from I, replacing element α with i_α. The sequence I is called *t-semistandard* if the elements of t do not decrease from left to right and downward, the "even" elements strictly increase along columns, and the "odd" elements strictly increase along rows.

The group $\mathfrak{S}_k$ naturally acts on sequences I. Let $\mathfrak{A}$ be the free supercommutative superalgebra with unit generated by $\{x_i\}_{i \in I}$. For any $\sigma \in \mathfrak{S}_k$, define $c(I,\sigma) = \pm 1$ from the equation

$$c(I,\sigma)x_I = x_{\sigma^{-1}I}, \quad \text{where } x_I = x_{i_1} \dots x_{i_k}. \tag{0.2}$$

Clearly, $c(I,\sigma)$ is a cocycle, that is,

$$c(I,\sigma\tau) = c(\sigma^{-1}I,\tau)c(I,\sigma).$$

With the help of this cocycle, a representation of $\mathfrak{S}_k$ in $T^k(V) = V^{\otimes k}$ for any superspace V may be defined as

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