Michigan Math. J. 49 (2001)

An Analog of the Classical Invariant Theory for Lie Superalgebras

Alexander Sergeev

This is a detailed version of my 1992 short announcement [S3]. For prerequisites on Lie superalgebras see Appendix 0 and Appendix 1, which are mainly borrowed from Leites's book [L3]. A draft of this paper was put on the net (math.RT/9810113) "earlier and independently", as Cheng and Wang referred to it in their papers [CW1; CW2], where they elucidate some of the results given here and also give an interpretation of a formula for projective symmetric functions. Still, a further elucidation will not hurt, and I intend to return to it elsewhere. Meanwhile, recall that Howe suggested a unified approach to the first and second theorems of the classical invariant theory: compare [Wy] with [H]. This approach becomes even more unified in [LSh], where Lie superalgebras that more or less implicitly linger in the background of [H] become the main characters. In this and a subsequent paper I consider analogs of these theorems for "classical" Lie superalgebras.

Related are problems on description of the centers of $U(\mathfrak{g})$ (cf. [LS; S1; S5]). The pioneer here was Berezin [B1; B2; B3], who somewhat differently considered, to an extent, $\mathfrak{g} = \mathfrak{gl}$, $\mathfrak{sl}$, and $\mathfrak{osp}$. Scheunert [Sch1; Sch2; Sch3; Sch4] has reproduced some of my results.

The reader should be aware of a totally different approach to invariant theory due to Shander [Sd1; Sd2], who justly observes that for Lie superalgebras it is possible not to restrict oneself to the study of polynomial functions and makes a step in this purely super direction.

1. Setting of the Problem. Formulation of the Results

1.0. Let V be a finite-dimensional superspace over $\mathbb{C}$ and let $\mathfrak{g}$ be an arbitrary *matrix* Lie superalgebra, that is, a Lie subsuperalgebra in $\mathfrak{gl}(V)$. Under the *classical invariant theory* for $\mathfrak{g}$ we mean the description of $\mathfrak{g}$-invariant elements of the algebra

$$\mathfrak{A}^{p,q}_{k,l} = S^{\boldsymbol{\cdot}}(V^p \oplus \Pi(V)^q \oplus V^{*k} \oplus \Pi(V)^{*l}),$$

where V^p denotes the direct sum of p copies of V. Clearly,

$$\mathfrak{A}^{p,q}_{k,l} = S^{\boldsymbol{\cdot}}(U \otimes V \bigoplus V^* \otimes W),$$

where $\dim U = (p, q)$ and $\dim W = (k, l)$. Therefore, Lie superalgebras $\mathfrak{gl}(U)$ and $\mathfrak{gl}(W)$ also act on $\mathfrak{A}^{p,q}_{k,l}$; hence, the enveloping algebra $U(\mathfrak{gl}(U \otimes W))$ also

Received August 2, 2000. Revision received January 22, 2001.